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On the deformed Einstein equations and quantum black holes

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Abstract. Recently q -deformed Einstein equations have been studied for extremal quantum black holes which have been proposed to obey deformed statistics by Strominger. In this study, we give the solutions of deformed Einstein equations by considering these equations for the charged black holes. Also we present the implications of the solutions, such as the deformation parameters lead the charged black holes to have a smaller mass than the classical Reissner-Nordström black holes. The reduction in mass of a classical black hole can be viewed as a transition from classical to quantum black hole regime.

1. Introduction

Recently, we have studied the q -deformed Einstein equations [1] for describing the gravitational fields of the Strominger's extremal quantum black holes which obey the deformed statistics [2]. Here, Verlinde's entropic gravity approach [3] have been used to obtain the q -deformed Einstein equations which are the gravitational field equations of charged quantum black holes with possible minimal mass. A charged black hole loses its mass via Hawking radiation and reaches a minimum mass proportional to the charge. The resulting black hole with a minimal mass is considered to be a micro black hole. Since these black holes can be considered as deformed bosons or deformed fermions, the micro black holes defined by q -deformed Einstein equations are assumed be the Ubriaco's q -deformed bosons [4].

We first give a brief summary of obtaining the q -deformed Einstein equations then obtain the solutions of the deformed Einstein equations for charged black holes. Since the solutions of standard Einstein equations for charged black holes are the Reissner-Nordström solutions in classical gravity, the solutions of the deformed Einstein equations for charged black holes can be considered in quantum gravity. Lastly the implications of the solutions are represented. These are that the deformation parameters lead the charged black holes to have a smaller mass than the usual Reissner-Nordström black holes. This reduction in mass of a usual black hole can be considered as a transition from classical to quantum black hole regime.

2. q -Deformed Einstein equations

We obtain the q -deformed Einstein equations which describe the gravitational fields quantum black holes by using the entropy of the Ubriaco's q -deformed Bose gas models in Verlinde's entropic gravity approach. The quantum algebraic structure of this q -deformed Bose gas model is given by the q -deformed boson algebra [4]

$$a_2 a_2^* - q^2 a_2^* a_2 = 1, \quad (1)$$



$$a_1 a_1^* - q^2 a_1^* a_1 = 1 + (q^2 - 1) a_2^* a_2, \quad (2)$$

$$a_2 a_1 = q a_1 a_2, \quad (3)$$

$$a_2 a_1^* = q a_1^* a_2. \quad (4)$$

Here a and a^* represents the deformed annihilation and creation operators, respectively. q is also a real deformation parameter with $0 \leq q < \infty$. The grand partition function of the q -deformed boson model is [4]

$$Z = \prod_k \sum_{m=0}^{\infty} (m+1) e^{-\beta \varepsilon_k \{m\}} z^m, \quad (5)$$

where $\beta = 1/kT$ and k is the Boltzmann constant, $z = e^{\beta\mu}$ is the fugacity, ε_k is the energy of the single-particle state, m is the occupation number of the single-particle state, and $\{m\}$ is the deformed occupation number and given by

$$\{m\} = \frac{1 - q^{2m}}{1 - q^2}. \quad (6)$$

The deformed entropy of the model is also given as

$$S = \frac{4\pi V (2m)^{3/2}}{h^3 T} E^{5/2} \left[\frac{5\sqrt{\pi}}{4} z + \frac{5\sqrt{\pi}}{2} \delta(q) z^2 - \frac{\sqrt{\pi}}{2} z \ln z - 2\sqrt{\pi} \delta(q) z^2 \ln z + \dots \right], \quad (7)$$

where $E = kT$ is the average energy of single particle, V is the volume enclosed by the deformed bosons, m is the mass of deformed bosons, T is the temperature of the model and $\delta(q) = (1/4) \cdot \{[3/(1+q^2)^{3/2}] - (1/\sqrt{2})\}$ [4]. The deformed entropy in (7) is used to obtain the one-parameter deformed or equivalently the q -deformed Einstein equations for q -deformed bosons.

We apply the Verlinde's proposal to the q -deformed Bose gas models, in order to construct the q -deformed Einstein equations from the entropy in (7). The fundamental notion needed to derive the gravity is information in the Verlinde's proposal. It is formally the amount of information associated with the matter and its location, measured in terms of entropy. When matter is displaced in space due to a reason, the result is a change in the entropy and this change causes a reaction force. This force is the gravity being an entropic force as an inertial reaction against the force causing the increase of the entropy [3].

The source of gravity is energy or matter and it is distributed evenly over the degrees of freedom in spacetime. The existence of energy or matter in spacetime causes a temperature in the spacetime. The product of the change of entropy during the displacement of source and the temperature is in fact the work and this work is originally led by the force which is known to be gravity [3].

By using the Verlinde's idea, q -deformed Einstein equations have recently been derived from the deformed entropy (7) of the q -deformed Bose gas model [1]. Eventually the q -deformed Einstein equations is given, as [1]

$$\frac{10\pi V (2mE)^{3/2}}{h^3} g(z, q) \left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \right) = 8\pi G T_{\mu\nu}, \quad (8)$$

where

$$g(z, q) = \left[\frac{5\sqrt{\pi}}{4} z + \frac{5\sqrt{\pi}}{2} \delta(q) z^2 - \frac{\sqrt{\pi}}{2} z \ln z - 2\sqrt{\pi} \delta(q) z^2 \ln z + \dots \right]. \quad (9)$$

By rewriting (8), we get

$$\Psi^q \left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \right) = 8\pi G T_{\mu\nu}, \quad (10)$$

where $\Psi^q = 10\pi V(2mE)^{3/2} g(z, q) / h^3$.

The equation (10) is the q -deformed Einstein equations, and it is assumed to describe the gravitational fields generated by the extremal quantum black holes which obey the statistics of deformed particles in accordance with the Strominger's proposal.

In the next section, we solve the deformed Einstein equations for a charged extremal black hole, and investigate the implications of the solutions.

3. Solution of the q -deformed Einstein equations

Deformed Einstein field equations will be used to describe the geometry of the spacetime around a charged and spherically symmetric quantum black hole. Therefore we need to solve the deformed Einstein-Maxwell equations due to the charge of the quantum black holes. Also, because of the spherical symmetry the metric has to be in the form for 4-dimension is [5]

$$ds^2 = -e^{2\alpha(r,t)} dt^2 + e^{2\beta(r,t)} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2. \quad (11)$$

The energy-momentum tensor $T_{\mu\nu}$ in (10) is

$$T_{\mu\nu} = F_{\mu\rho} F_{\nu}^{\rho} - \frac{1}{4} g_{\mu\nu} F_{\rho\sigma} F^{\rho\sigma}, \quad (12)$$

where $F_{\mu\nu}$ is the electromagnetic field strength tensor and the trace of $T_{\mu\nu}$ is zero due to the electromagnetic strength tensor. Taking the trace of (30) leads to

$$\Psi^q R_{\mu\nu} = 8\pi G T_{\mu\nu}. \quad (13)$$

Spherical symmetry and absence of magnetic charge for our quantum black hole gives the electromagnetic field strength tensor has only radial electric field components $E_r = F_{tr} = -F_{rt} = f(r, t)$.

By equating the non-zero components of the Ricci tensor for the metric (11) and the corresponding non-zero components of the energy-momentum tensor obtained by (12), we find $\alpha(r, t) = \alpha(r) = -\beta(r)$. The solution of the Maxwell equations $g^{\mu\nu} \nabla_{\mu} F_{\nu\sigma} = 0$ and $\nabla_{[\mu} F_{\nu\rho]} = 0$ gives $f(r, t) = f(r) = Q / \sqrt{4\pi} r^2$. Using this electromagnetic field strength tensor for one of the components of the Ricci tensor and the corresponding energy-momentum tensor in (13) gives the solutions of the deformed Einstein equations:

$$ds^2 = \Delta dt^2 + \Delta^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2, \quad (14)$$

where

$$\Delta = 1 - \frac{2Gm}{r} + \frac{1}{\Psi^q} \frac{GQ^2}{r^2}. \quad (15)$$

The singularities and the event horizons for these black holes are determined by the function Δ and the radius r . There is a true curvature singularity at $r = 0$, since the metric goes to infinity for this value. The coordinate singularity also occurs at $\Delta = 0$ and the conditions giving this singularity occur from the solution of $\Delta = 0$, such as

$$r_{\pm} = Gm \pm \sqrt{G^2 m^2 - \frac{GQ^2}{\Psi^q}}. \quad (16)$$

The case $Gm^2 = Q^2 / \Psi^q$ stands for the quantum charged black hole case, since the mass of the black hole decreases to the minimum value from the case $Gm^2 > Q^2 / \Psi^q$. A minimum mass solution for the quantum black holes is reached by the Hawking radiation and remains stationary. $Gm^2 = Q^2 / \Psi^q$ case makes $\Delta = 0$ at a single radius $r_{\pm} = Gm$ and refers to a single event horizon. This deformed

case solution $Gm^2 = Q^2 / \Psi^q$ is the analogue of classical Reissner-Nordström solution $m = Q / \sqrt{G}$ which is often examined in the studies of quantum gravity. For $Gm^2 > Q^2 / \Psi^q$ case, the mass of the black hole is allowed to be in very large classical scales due to the ability of getting bigger values than the charge. Whereas the mass of the deformed black hole is allowed to decrease very small values which could fall into the quantum regime, because the decrease of the mass is governed by a very small term $1 / \Psi^q$ being order of $h^{6/7}$.

Mass reduction with respect to the classical Reissner-Nordström case can be compared for q -deformed cases. Using $1 / \Psi^q$ from (10) gives the mass of the extremal quantum black hole in case of $Gm^2 = Q^2 / \Psi^q$, such as

$$m^q = \left(\frac{h^3}{10\pi V(2E)^{3/2}} \frac{1}{g(z, q)} \right)^{\frac{2}{7}} \left(\frac{Q^2}{G} \right)^{\frac{2}{7}}, \quad (17)$$

The comparison of the deformed black hole mass and classical Reissner-Nordström black hole mass $m = Q / \sqrt{G}$ reads as

$$m^q = \left(\frac{1}{10\pi V(2E)^{3/2} Q^{3/2}} \right)^{\frac{2}{7}} \left(\frac{h^3 G^{3/4}}{g(z, q)} \right)^{\frac{2}{7}} m, \quad (18)$$

The equation in (18) implies that the mass of the charged extremal black hole in the deformed quantum case can decrease to a smaller value than that of the classical Reissner-Nordström case. We figure out the decrease in the mass by examining the behaviours of the factor $(h^3 G^{3/4} / g(z, q))^{2/7}$ in the classical mass of the charged black hole in (18). We illustrate the behaviour of $(h^3 G^{3/4} / g(z, q))^{2/7}$ with respect to z and q in Figure 1 and Figure 2, for $q < 1$ and $1 < q$, respectively.

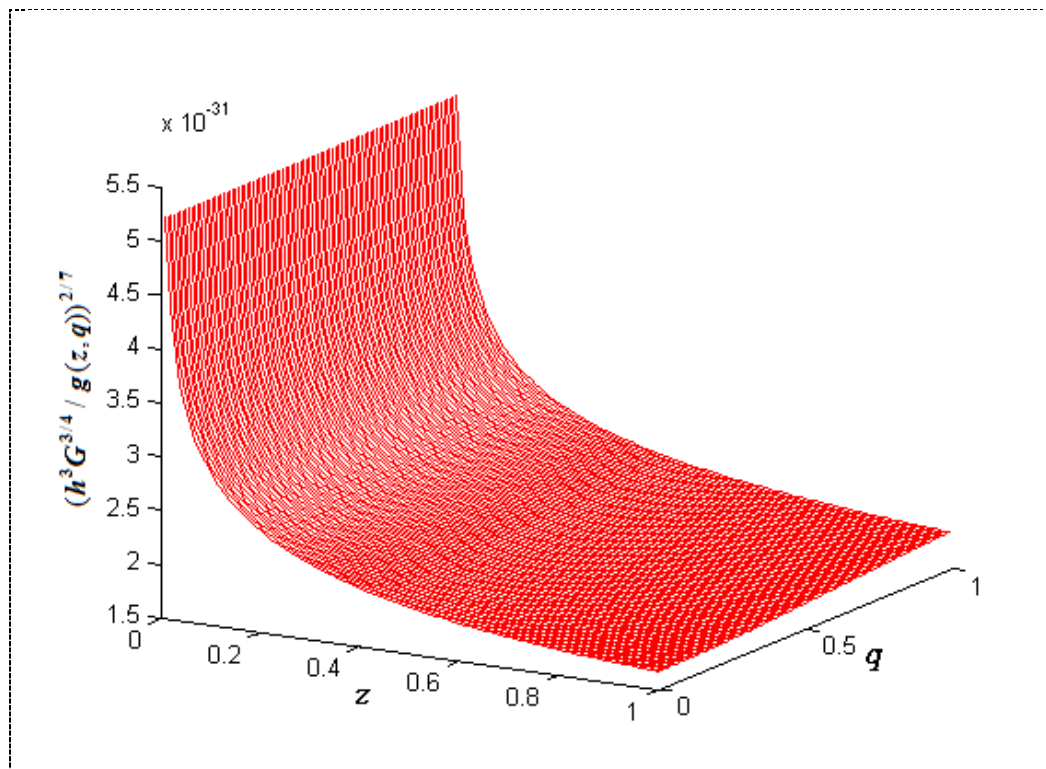


Figure 1. Mass reduction factor $(h^3 G^{3/4} / g(z, q))^{2/7}$ for various values of the deformation parameter $q < 1$.

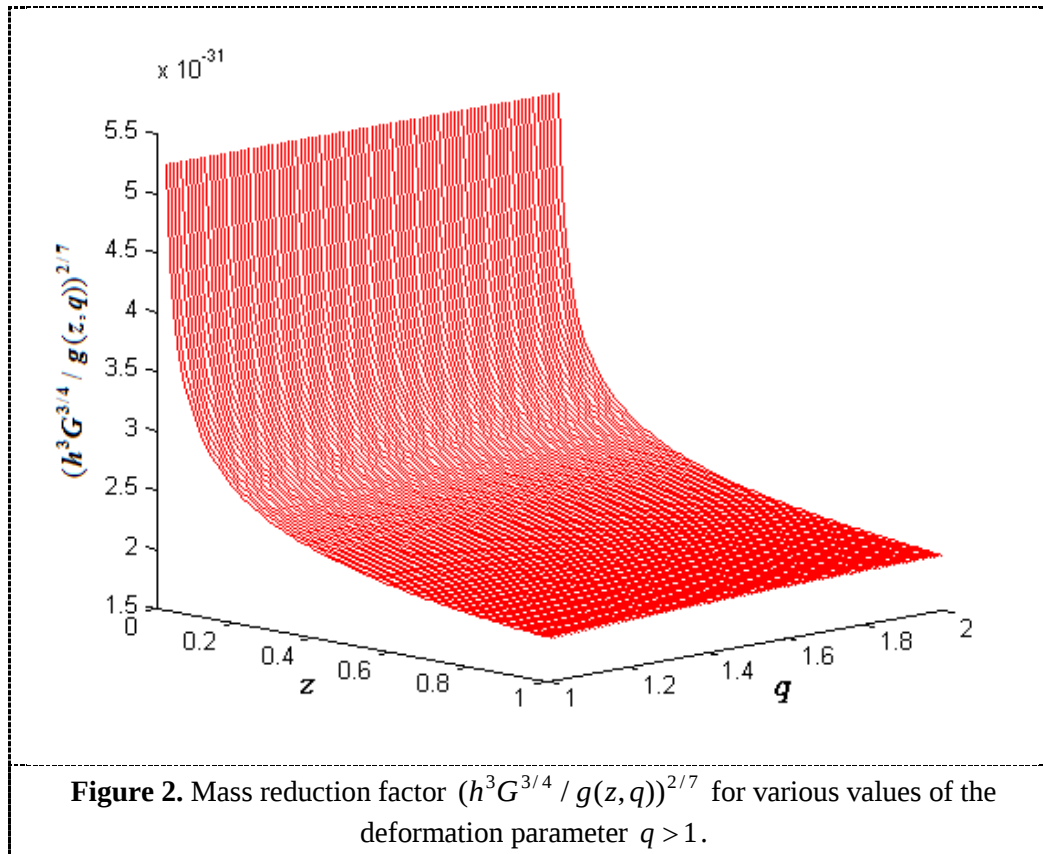


Figure 2. Mass reduction factor $(h^3 G^{3/4} / g(z, q))^{2/7}$ for various values of the deformation parameter $q > 1$.

4. Conclusions

In this study, we review the q -deformed statistics of bosons and obtained the q -deformed Einstein equations, which is based on the Strominger's idea, such that the micro quantum black holes obey the deformed statistics. We then obtain the solutions of q -deformed Einstein equations for the charged extremal quantum black holes. We then interpret the solutions for q -deformed Einstein equations.

There occur two singularities from the solutions of deformed Einstein equations, such that the true and coordinate singularities. The possible decrease in mass via Hawking radiation to the minimum value which is determined by the charge of quantum black hole is investigated and the difference between the classical black holes and quantum black holes is compared from the equations (18). We represent the decrease in quantum mass m^q in Figure 1 and Figure 2. According to the Figures 1 and 2, the mass of the quantum black hole m^q in (18) is at least 10^{-30} times smaller than the classical black hole mass m , in the deformed case, with the inverse of the volume, charge and energy factors, mass m^q gets smaller than $10^{-30} m$.

Three independent studies, Strominger's micro black holes obeying the deformed statistics, Verlinde's entropic gravity approach and the thermostatics of deformed Bose gas model, seem to be consistent with each other, since the theoretical possibility of concentrating a mass into its reduced Planck mass leads to a quantum black hole whose dynamics described by the deformed Einstein equations and the mass reduction to a smaller value than that of the classical black holes.

5. References

- [1] Dil E 2015 *Canad. J. Phys.* **93** 1274
- [2] Strominger A 1993 *Phys. Rev. Lett.* **71** 3397
- [3] Verlinde E 2011 *J. High Energy Phys.* JHEP04(2011)029
- [4] Ubriaco M R 1997 *Phys. Rev. E* **55(1)** 291
- [5] Wald R M 1984 *General Relativity* (Chicago: The University of Chicago Press)