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A study on structural evolution of $^{142-164}$Nd isotopes

Seyit O. Kara[1,*], Tuncay Bayram[2], Serkan Akkoyun[3]

[1] Bor Vocational School, Nigde University, Nigde, Turkey
[2] Department of Physics, Sinop University, Sinop, Turkey
[3] Vocational School of Health Services, Cumhuriyet University, Sivas, Turkey

E-mail: *seyitokankara@gmail.com

Abstract. Constrained Hartree-Fock-Bogoliubov theory with SLy5 Skyrme force has been applied for even-even $^{142-164}$Nd isotopes to investigate the structural evolution of Nd isotopic chain. In this work, ground-state energies and charge radii of Nd isotopes have been carried out as in good agreement with the experimental data. The systematic investigation of ground-state shape evolution between spherical U(5) and axially deformed SU(3) for $^{142-164}$Nd has been studied by using potential energy curves.

1. Introduction

Nuclear models can describe main features of nuclei along the nuclear chart while a full microscopic description of the nuclear many-body system is often needed to describe the full variety of nuclear behaviours. In particular, the size and shape of nuclei are very important to describe their behaviours and different regions of nuclidic chart can be characterized by using macroscopic concept of nuclear shape (spherical nuclei, axially deformed nuclei, gamma unstable nuclei, etc). Nuclei can exhibit a change of their shapes along the isotopic or isotonic chains based on changes of their neutron or proton numbers. This refers to Quantum Phase Transition (QPT) in shape of nuclei. The Interacting Boson Model (IBM) [1] has three phases, with the U(5), SU(3) and O(6) dynamical symmetries corresponding to the breaking of U(6) into its sub-algebras. U(5), SU(3) and O(6) symmetries are associated with spherical, axially deformed and γ-unstable shapes of nuclei, respectively. More recently, Iachello has proposed the critical-point symmetries X(5) and E(5) which corresponds to the first order [2] and second order [3] phase transition, respectively. The symmetry X(5) corresponds to the critical-point between the U(5) and SU(3) dynamical symmetry limits while the E(5) symmetry characterizes the phase transition region between the U(5) and O(6) dynamical symmetries. Experimental evidences of the X(5) and E(5) critical-point symmetries have been found in the spectrum of ^{152}Sm [4] and ^{134}Ba [5], respectively. Because of these progresses, theoretical and experimental investigations of nuclei with possible critical-point symmetry have been a hot topic in recent decade.

Bohr collective Hamiltonian and the IBM are used for investigating shape phase transitions in nuclei. On the other hand, Hartree-Fock-Bogoliubov (HFB) method [6, 7] and relativistic mean field (RMF) model [8, 9] successfully have been applied to search possible critical-point nuclei [10, 11, 12, 13]. Generally, structural evolution of an isotopic chain has been used for determining possible critical-point nuclei by using calculated potential energy curves (PECs) of nuclei in these studies. Many nuclei have been pointed out as possible critical-point nuclei with E(5) and X(5) symmetry within the framework of RMF theory and HFB method. A brief


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summary on studies for determining of possible critical-point nuclei within the mean field models can be found in [11, 13] and references therein.

Various quantities such as two-neutron separation energies S_{2n}, various electric quadrupole transition rates ($B(E2;2_1^+ \rightarrow 0_1^+)$ and $B(E0;0_2^+ \rightarrow 0_1^+)$), isomer shifts, isotope shift and the characteristic ratio $R_{4/2} = E(4_1^+)/E(2_1^+)$ of nuclei are used to search critical-point nuclei. The $N = 90$ isotones of Nd, Sm, Gd and Dy are associated with X(5) symmetry. After the first experimental identification for X(5) behavior of ^{152}Sm, ^{150}Nd was identified as exhibiting X(5) behavior [14]. Additionally, ^{148}Nd has been suggested as an example of nucleus with X(5) critical-point symmetry via an analytic solution using β^2 potential [15]. Furthermore, the PECs of Nd isotopic chain has been carried out within the framework of axially deformed RMF theory [10]. In this study, authors have concluded that the PECs for nuclei suggested as good examples of X(5) exhibit a bump by using chains of isotopes involving nuclei suggested to be good examples of the X(5) critical-point symmetry. In the present study, the ground-state properties of even-even $^{142-164}$Nd isotopes have been carried out by using HFB method. The structural evolution of $^{142-164}$Nd isotopes has been discussed by examining their PECs obtained by imposing a constraint on the mass quadrupole moment for Nd isotopes. In addition, some signatures of X(5) symmetry such as two-proton separation energies, the differences between squares of ground-state charge radii and proton-neutron interaction strength δV_{pn} of $^{142-164}$Nd isotopes have been investigated.

2. HFB method and details of calculations

In HFB method, a two-body Hamiltonian of a system of fermions in terms of a set of annihilation and creation operators (c, $c^\dagger$) has the form:

$$H = \sum_{n_1 n_2} e_{n_1 n_2} c^\dagger_{n_1} c_{n_2} + \frac{1}{4} \sum_{n_1 n_2 n_3 n_4} \bar{v}_{n_1 n_2 n_3 n_4} c^\dagger_{n_1} c^\dagger_{n_2} c_{n_4} c_{n_3} \,, \tag{1}$$

where $\bar{v}_{n_1 n_2 n_3 n_4} = \langle n_1 n_2 \mid V \mid n_3 n_4 - n_4 n_3 \rangle$ are anti-symmetrized matrix elements of the two-body effective nucleon-nucleon interaction. In terms of Skyrme forces, the total Hamiltonian is the sum of the mean-field and pairing energy densities. In the present study, a parametric form of total HFB energy with the Skyrme force SLy5 [16] has been used as in Ref. [17]. Tensor effect has been taken into account. The code HFBTHO (v1.66p) [17] has been used to reproduce ground-state properties of even−even $^{142-164}$Nd isotopes which utilizes an axially symmetric deformed harmonic oscillator potential. The PECs of even−even isotopic chain of Nd have been obtained by utilizing quadrupole moment constraint as prescribed in Ref. [17]. The number of oscillator shells taken into account was 16 and good convergence were obtained in the calculations.

3. Results and discussions

Several nuclear models which can be mainly classified into three categories (macroscopic, macroscopic-microscopic and microscopic models) have been successfully employed to carry out masses of nuclei through the nuclear chart. A brief summary on the studies for mass predictions with nuclear models can be found in [18] and references therein. As a microscopic model, HFB method with Skyrme type forces [16] can be applied in the whole mass region along the nuclear chart to reproduce ground-state energies, sizes and deformations of nuclei [19, 20]. Calculated ground-state binding energies (BEs) of even-even $^{142-164}$Nd obtained from this work are listed in Table 1. The predictions of Finite Range Droplet Model (FRDM) [21] and RMF model with NL3* functional [18] and experimental data [22] are also listed for comparison. As can be understood from Table 1, the predictions of FRDM whose parameters are fitted by using many experimental masses of nuclei are closer to the experimental data rather than those of HFB and RMF. On the other hand, it can be say that HFB and RMF predictions for BEs of Nd isotopes are reliable. Two-neutron separation energy S_{2n} which can be derived easily from the relation

Table 1. Ground-state BEs of $^{142-164}$Nd isotopes (in units of MeV) obtained from this work by using HFB method with SLy5 Skyrme force. The predictions of FRDM and RMF model with NL3* functional and experimental data are also listed for comparison.

Nuclei	FRDM [21]	RMF [18]	This work	Experiment [22]
^{142}Nd	1185.67	1188.83	1182.84	1185.14
^{144}Nd	1199.16	1200.97	1194.26	1199.01
^{146}Nd	1212.00	1210.69	1207.42	1212.40
^{148}Nd	1225.33	1223.38	1220.66	1225.03
^{150}Nd	1238.46	1235.15	1233.24	1237.45
^{152}Nd	1250.93	1247.88	1246.17	1250.06
^{154}Nd	1262.45	1258.96	1257.24	1261.74
^{156}Nd	1273.02	1269.10	1267.17	1272.71
^{158}Nd	1282.88	1278.82	1276.88	1282.80
^{160}Nd	1292.14	1287.89	1286.48	1291.84
^{162}Nd	1300.81	1295.60	1293.72	
^{164}Nd	1308.31	1302.27	1300.56	

$$S_{2n} = BE(N, Z) - BE(N-2, Z) \tag{2}$$

is important quantity to expose the shell structure of nuclei. In this equation (2), BE is the total binding energy of the nuclei with N neutrons and Z protons. Figure 1 shows the calculated S_{2n} values of even-even $^{142-164}$Nd isotopes obtained by using HFB method with the SLy5 parameters. The available experimental data and the predictions of FRDM and RMF model are also shown for comparison. As can be seen in Figure 1, except the neutron number $N = 84$, our calculations and the predictions of RMF model are in good agreement with the experimental data. An abrupt decrease in S_{2n} at magic neutron number $N = 82$ corresponds to the closed shell.

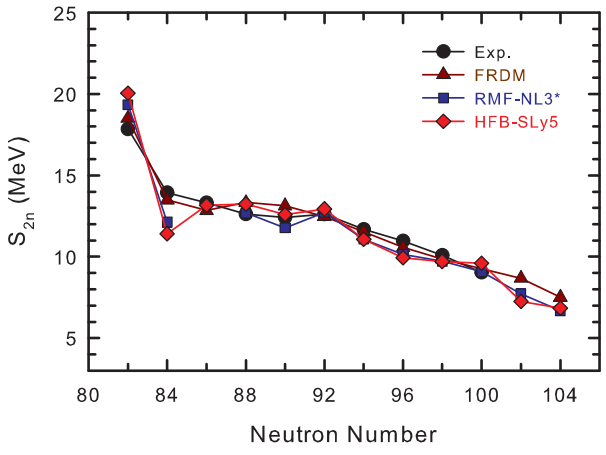


Figure 1. Comparison of the calculated two-neutron separation energies (S_{2n}) of $^{142-164}$Nd with the RMF model, FRDM and experimental results.

The concept of critical point symmetry includes analytic expressions for observables such as transition rates B(E2) and excitation energies. These observables of a nucleus have to be calculated to indicate that it is a critical-point nucleus quantitatively and this is not possible on the mean-field level. On the other hand, relativistic and non-relativistic mean field models have

been employed in calculations of potential energy curves (PECs) as functions of the quadrupole deformation parameter β for isotopic chains in which the occurrence of critical point symmetries had been predicted [10, 12]. The resulting PECs display shape transitions from spherical to deformed configurations. In these studies relatively flat PECs over an extended range of the deformation parameter have been obtained in nuclei showing the E(5) symmetry, while in nuclei showing the X(5) symmetry, PECs with a bump have been found. It should be noted, however, that these assumptions are reliable qualitatively.

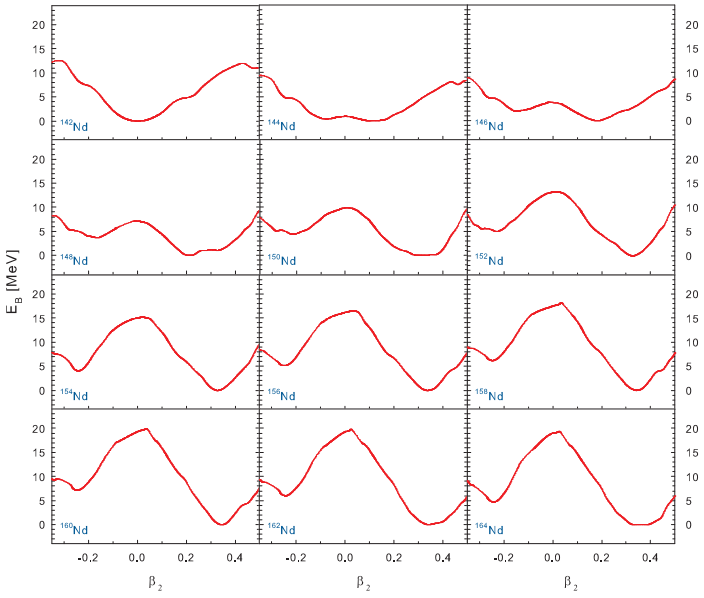


Figure 2. The PECs for $^{142-164}$Nd obtained from HFB method with the Skyrme force SLy5.

In Figure 2, the PECs of even-even $^{142-164}$Nd as a function of quadrupole moment deformation parameter β obtained from HFB calculations with the Skyrme force SLy5 are shown. The PECs as function of β deformation has been obtained by imposing a constraint on the mass quadrupole moment for Nd nuclei. The particle number is only approximately restored by the Lipkin-Nogami procedure. The BE of each Nd isotope for the ground-state is taken as reference in these PECs. As can be clearly seen from Figure 2, the PECs exhibit a gradual transition between the spherical ^{142}Nd and well deformed prolate ^{164}Nd. In particular, a wide flat minimum on the prolate side and additional 4 MeV excitation energy on the oblate side of the PEC of ^{150}Nd are found and the potential barrier between two minima is seen as being approximately 10 MeV. Because of the assumption of Ref. [10] that the PECs found for nuclei suggested as good examples of X(5) exhibit a bump, ^{164}Nd could be indicated as a possible critical-point nucleus with X(5) symmetry. Furthermore, it should be noted that the similarity between the projections on the prolate and oblate axes of the original X(5) potential considered by Iachello [2] and the calculated PEC of ^{150}Nd shown in Figure 2 is notable.

In the present study, the differences between squares of ground-state charge radii of Nd isotopes with neutron number N and those of the reference nucleus ($N = 82$) have been calculated for better understanding the structural evolution of Nd isotopes with increasing neutron number. Calculations obtained from HFB method with SLy5 parameters and available experimental data [23] are shown in Figure 3. As can be seen in Figure 3, after three steps rising in neutron numbers $N \leq 88$ an abrupt change starts at $N \leq 90$ ($A = 150$) and then the nuclei remain in deformed shape. This is a signature of phase-transitional behavior of ^{150}Nd.

In addition, first order phase transitional behavior in the equilibrium deformation has further highlighted the key role of the proton-neutron (p-n) interaction. Direct correlation between

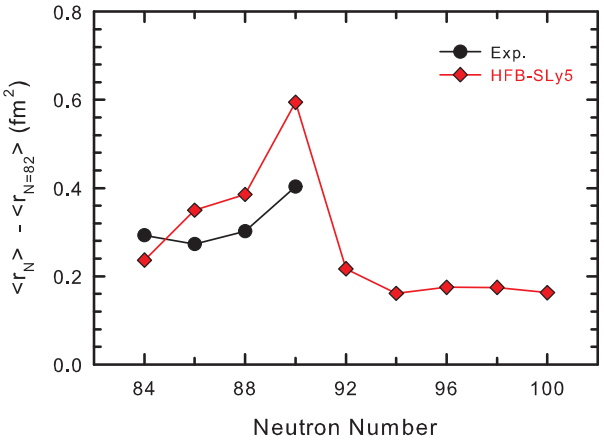


Figure 3. Calculated differences between squares of ground-state charge radii: $\langle r_N^2 \rangle - \langle r_{N=82}^2 \rangle$ as functions of neutron number in Nd isotopes.

observed growth rates of collectivity and empirical p-n interaction strengths have been found by Cakirli and Casten [24]. Because of this, we have calculated p-n interaction strengths of $^{142-164}$Nd. The average p-n interaction strength for an even-even nucleus can be calculated by using the formula [25]

$$\mid \delta V_{pn}^{ee}(Z,N) \mid = \frac{1}{4}[(B_{Z,N} - B_{Z,N-2}) - (B_{Z-2,N} - B_{Z-2,N-2})]. \quad (3)$$

The calculated and available experimental data [22] for δV_{pn} of even-even $^{142-164}$Nd are shown in Figure 4. As can be seen from this figure, the calculated values of δV_{pn} for $^{142-164}$Nd are in agreement with the experimental data. An abrupt increase in δV_{pn} starts at neutron number $N = 90$ ($A = 150$) which means that collectivity of nuclei grows in this isotopic chain and this may be considered as a signature of shape phase transitional character of ^{150}Nd from spherical to axially deformed shape.

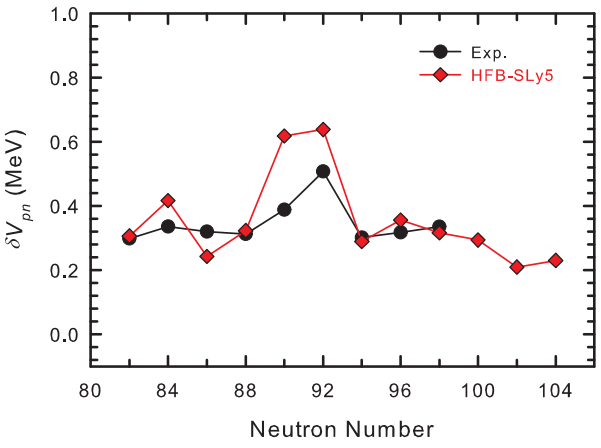


Figure 4. The calculated and available experimental δV_{pn} for $^{142-164}$Nd.

4. Conclusions

Axially deformed HFB calculations with the SLy5 Skyrme force have been carried out to investigate structural evolution of ground-state of even-even $^{142-164}$Nd. The calculated ground-state BEs and S_{2n} of Nd isotopes have been found in good agreement with the experimental data.

By examining the PECs of Nd isotopic chain, ^{150}Nd has been indicated as a possible critical-point nucleus with X(5) symmetry that is favored by the experimental data. Furthermore, the same conclusion of the transition has been supported by investigating differences in the ground-state charge radii of Nd isotopes.

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