



Original Article

Neutronics modeling of bubbles in bubbly flow regime in boiling water reactors

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ABSTRACT

This study mainly focused on the neutronics modeling of bubbles in bubbly flow in boiling water reactors. The bubble, ring and homogenous models were used for radial void fraction distribution. Effect of the bubble and ring models on the infinite multiplication factor and two-group flux distribution was investigated by comparing with the homogenous model. Square pitch unit cell geometry was used in the calculations. In the bubble model, spherical and non-spherical bubbles at random positions, sizes and shapes were produced by Monte Carlo method. The results show that there are significant differences among the proposed models from the viewpoint of physical interaction mechanism. For the fully-developed bubbly flow, k_{inf} is overestimated in the ring model by about 720 ± 6 pcm with respect to homogeneous model whereas underestimated in the bubble model by about -65 ± 9 pcm with a standard deviation of 15 pcm. In addition, the ring model shows that the coolant must be separated into regions to properly represent the radial void distribution. Deviations in flux distributions principally occur in certain regions, such as corners. As a result, the bubble model in modeling the void fraction can be used in nuclear engineering calculations.

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1. Introduction

Bubbles formed during the boiling and their distribution in axial and radial directions are expressed in terms of void fraction distributions. To effectively use them in neutronic and thermal-hydraulic calculations, a number of experiments were performed by the researchers in various channels including rod bundle geometries in order to comprehend the shapes, velocities and distributions of bubbles [1–3]. Bubble-layer thickness was successfully introduced into the void fraction [4]. Distribution parameter and drift velocity constitutive equations were obtained for sub-cooled boiling flow in a sub-channel of rod bundle geometry [5], as well. The rod wall and sub-channel geometry effects to the constitutive equation of drift velocity were added. Even analytical models for steady, axisymmetric and fully developed adiabatic flows with two fluid model were tried to develop [6]. But, Hosakawa et al. [1], who experimentally measured the distributions of local void fraction and bubble velocity in each sub-channel in a 4×4 rod bundle using a double-sensor conductivity probe,

states that the distributions of void fraction in inner and side sub-channels depend not only on the lift force acting on bubbles but also on the geometrical constraints on bubble dynamics, i.e. the effects of rod wall on bubble shape and rise velocity. In this context, it has been noticed that there are significant differences between the models of void fraction distributions (e.g. uniform or varying). As an illustration, for BWR 8×8 fuel bundles at about 40% void fraction, a maximum decrease of about 150 pcm in the k_{inf} is calculated due to the presence of Gd rods and control rods leading to the void heterogeneity [7]. Moreover, the results obtained by using THERMIT2 and SRAC95 codes in a fuel assembly show that there is a maximum difference of about 60 pcm in k_{inf} when the uniform void fraction distribution is compared with sub-channel-wise void distribution [8]. The effects of using a non-uniform sub-channel moderator density for the sub-channel of 11×11 ATRIUM fuel assembly were discussed in a different work [9]. The difference in k_{inf} at the beginning of cycle was found to be 806 pcm due to both high gadolinium concentration and the control blade. By using CASMO-4, HELIOS/PRESTO-2 and MCNP4C, Jatuff et al. [10] found that the impact of the void fraction heterogeneity on the reactivity and pin power distribution can be as high as 200 pcm and 4% for BWR assembly of the type Westinghouse SVEA-96. In the

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NUPEC BWR full-size fine-mesh bundle test (BFBT) comparison analyzes for the understanding and modeling of two-phase flow behavior in real fuel bundles, uncertainty analyzes were carried out for the void distribution in the sub-channel and for the critic power phases [11].

From the neutronic modeling perspective, which also forms the subject of this study, void fraction representation may differ from modeling the boiling in the form of bubbles due to their unique interaction mechanisms and flow dynamics. Besides, neutronic modeling of bubbles, individually or together, is a quite difficult and complex task due to their physical properties, phase motions and geometric shapes. Yet, among the flow regimes (classified as sub-cooled, bubbly, slug, churn, and annular) observed in a typical BWR sub-channel, bubbly flow appears to be well understood and can be modelled by virtue of its relatively simple geometric structure, predictable flow behavior and relatively known void profile, i.e. bubbly flow consists mostly of bubbles with a spherical structure at various radii. They can be simulated inside a sub-channel with an appropriate void distribution. However, bubble modeling has not thus far been the subject of any work due to the aforementioned reasons. This study, therefore, deals with the creation of a physical (neutronic) modeling of bubbles for the bubbly flow. It is thus expected that the effects of the void fraction approaches (e.g. uniform distribution) involved in nuclear engineering calculations on reactor parameters would reveal.

2. Modeling and method

2.1. Design parameters

Square unit cell structure was selected for the bubble modeling because of its relative simplicity in geometry. Of the unit cell, geometric and material design parameters are from BWR PB-2 of the UAM comparative analysis [12] whereas thermal-hydraulic design parameters are from the BWR Turbine Trip (TT) benchmark study [13]. Moderator density is calculated from Eq. (1) due to void fraction dependency. The design parameters are given in Table 1.

2.2. Characteristics of bubbles in bubbly flow

In a two-phase flow, average phase density of a homogeneous mixture inside a control volume can be written in the form of Eq. (1). When the phase densities are assumed uniform in the volume, mixture density equals to

$$\langle \rho \rangle = \langle \alpha \rho_v \rangle + \langle (1 - \alpha) \rho_l \rangle = \alpha \rho_v + (1 - \alpha) \rho_l \quad (1)$$

It should be noted that bubbles in a sub-channel move continuously in the directions of r , θ and z . Therefore, the void fraction occupied by k -phase averaged over a specific control volume and time interval should be expressed by Eq. (2).

$$\langle \alpha \rangle = \frac{1}{V \Delta t^*} \int_{t-\Delta t^*/2}^{t+\Delta t^*/2} \iiint_V \alpha_k dV dt \quad (2)$$

However, radial void distribution becomes important in sub-channel boiling as bubbles accumulate at the center of the sub-channel as in BWR case. In the case of bubbly flow in a cylindrical tube, the distribution of bubbles over time and position bounded by the radius of the tube can be expressed by Eq. (3) [14].

$$\frac{\alpha_k - \alpha_{kw}}{\alpha_{k0} - \alpha_{kw}} = 1 - \left(\frac{r}{R_w} \right)^n \quad (3)$$

where 0 and w subscripts indicate the values of k -phase in the center and tube wall, respectively, R_w is the radius of the tube and r is the radial distance. Noting that the exponential power n depends on the flow type, the void fraction, the velocity of the bubbles in the liquid. However, a value of 7 for n is typically used for a fully-developed bubbly flow. When this expression is rearranged for a sub-channel (as a unit cell) illustrated in Fig. 1, radial void distribution, Eq. (4), becomes [5].

$$\alpha(r, \theta) = \alpha_{c0} \left[1 - \left\{ 1 - \frac{P_0 - 2R_0 \cos \theta}{(\sqrt{2}P_0 - 2R_0) \cos \theta} \right\}^n \right] \times \left\{ 1 - \left(1 - \frac{2r \cos \theta}{P_0 - 2R_0 \cos \theta} \right)^n \right\} \quad (4)$$

Where P_0 is the unit cell pitch, R_0 is outer radius of the fuel rod, θ is the angle between x -axis and ℓ , α_{c0} is the maximum void fraction

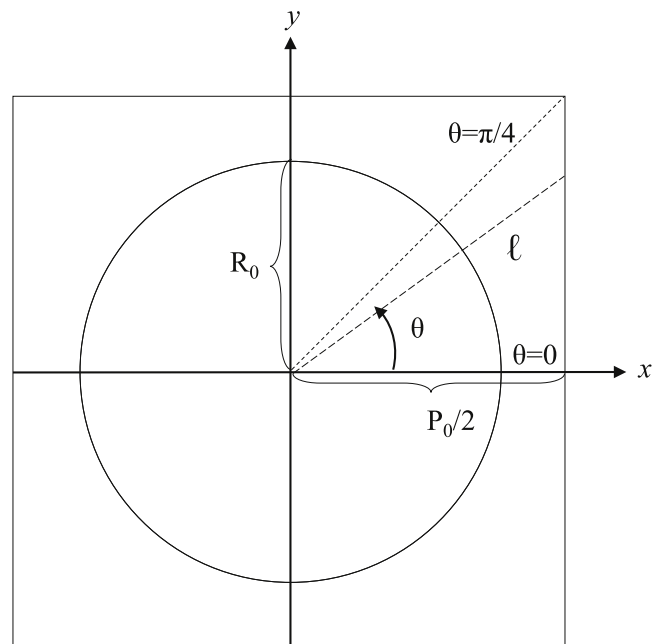


Fig. 1. Sub-channel geometry considered for bubble distribution.

Table 1
Design parameters of BWR PB-2 unit cell.

Parameter	Unit	Value
Pitch	mm	18.75
Fuel pellet diameter	mm	12.1158
Fuel pellet material		UO ₂
Fuel density	g/cm ³	10.42
Fissile content	w/o	2.93
Clad outside diameter	mm	14.3002
Clad thickness	mm	0.9398
Clad material		Zircaloy-2
Clad material density	g/cm ³	6.55
Gap material		He
Moderator/coolant material		H ₂ O
Operating Pressure	MPa	7.2
Fuel temperature	K	900
Clad temperature	K	600
Moderator/coolant temperature	K	557
Moderator/coolant density	kg/m ³	Eq. (1)

at the corner of the sub-channel and

$$R_0 \leq r \leq \frac{P_0}{2 \cos \theta}$$

Plots of the radial void distribution according to Eq. (4) are given in Fig. 2(a) for $n = 7$ and $\alpha = 15\%$ in the range of $\theta = 0$ and $\theta = \pi/4$ and in Fig. 2(b) for various variables.

It is possible to define an average bubble diameter $\bar{d}$, in the simplest form, for bubbly flow, given in Eq. (5) [15], by assuming a spherical shape. Bubbles are so small enough that surface tension forces make them spherical.

$$\bar{d} = \frac{\sqrt[4]{7.5d_{\max}}}{3} \quad (5)$$

According to the vertical flow regime map (Fig. 11.4) given in Todreas and Kazimi [16] for the BWR design tabulated in Table 1, void fraction of the bubbly flow is in the range of 0–0.25. The maximum diameter of the bubbles corresponding to this flow (Table 11.3) is $d_{\max} < 5$ mm. Inserting this information into Eq. (5) yields an average bubble diameter of about 3 mm. This value agrees very well with the experimental results [17] for small bubbles measured for bubbly flows.

In addition to the average bubble diameter, mean temperature of the bubble (T_b) required to maintain the continuity of a bubble with a radius of r_b at a constant temperature and a pressure (P_ℓ) needs to be determined from Eq. (6) [16].

$$T_b - T_{\text{sat}} = \frac{RT_b T_{\text{sat}}}{h_{fg}} \ln \left(1 + \frac{2\sigma}{P_\ell r_b} \right) \quad (6)$$

where σ is the surface tension, T_{sat} is the saturation temperature at the liquid pressure P_ℓ , R is the gas constant and h_{fg} is the specific enthalpy. Using the above information, the void distribution can be simulated in different ways: in the forms of homogenous, rings and bubbles.

2.3. Modeling of coolant density variation

2.3.1. Homogenous model

As a simplest approach, the homogenous model, widely used due to making the modeling easier, is based on modeling of the coolant/moderator at a uniform density in all direction. The density of the coolant is calculated by using Eq. (1) by preserving the average void fraction over a control volume (unit cell).

2.3.2. Ring model

In the ring model, the coolant is represented as rings (regions) to reflect the density variation in radial direction. To understand this modeling effect, single ring is selected as a reference calculation. Remember that defining the coolant as a single region also corresponds to the homogenous model. The rings are of equal thickness. No criterion is applied to increase the number of rings. A representation of the model is shown in Fig. 3. The thickness of the rings is either fixed (as in this study) or equal-volume or determined from the gradient-based method. It is expected that other methods would also yield the same results after a certain number of regions. Density of each ring is calculated by using Eq. (7)

$$\langle \alpha \rangle_i = \frac{\int_{R_1}^{R_2} \int_{\theta}^{\pi/4} \alpha(r, \theta) r dr d\theta}{\int_{R_1}^{R_2} \int_{\theta}^{\pi/4} r dr d\theta} \quad (7)$$

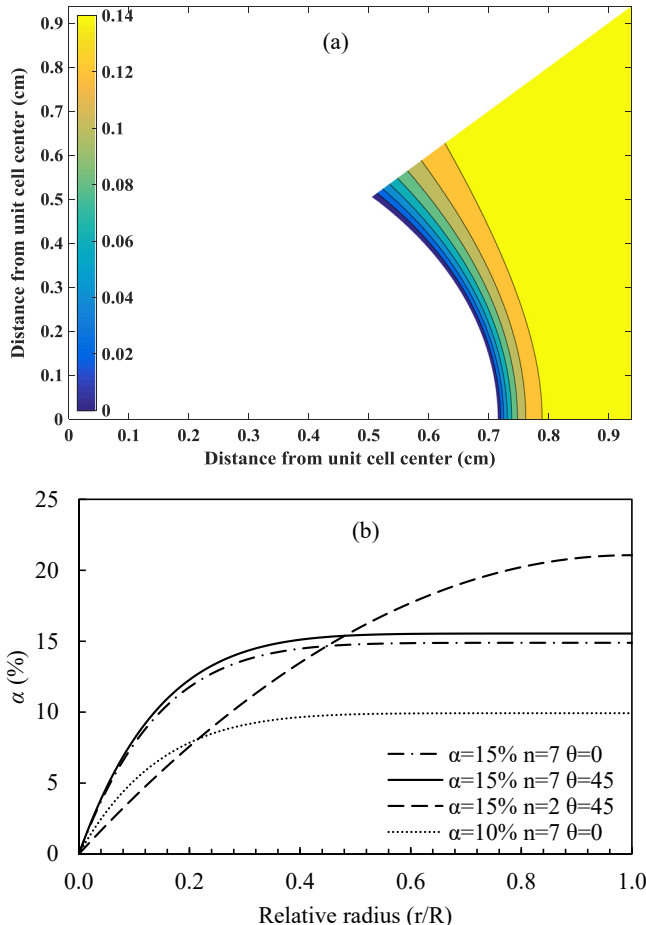


Fig. 2. Radial void distribution (a) at one-eighth of the unit cell (b) for various variables.

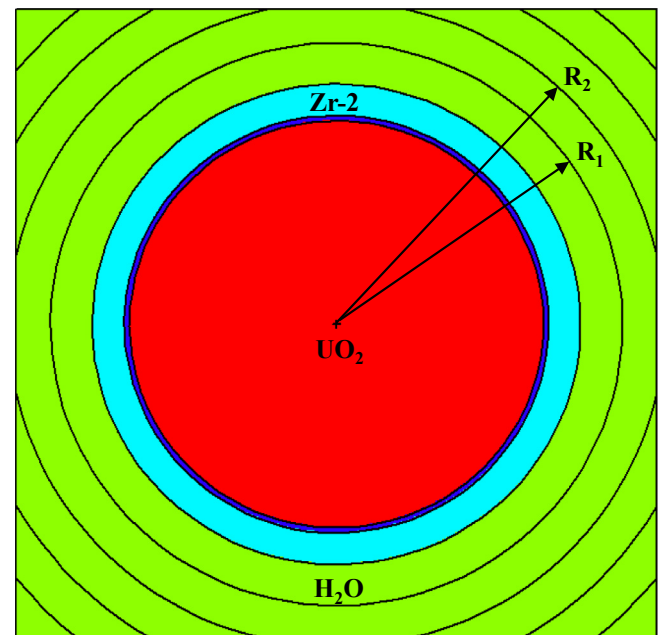


Fig. 3. Ring model for the coolant region.

where $\theta = \cos^{-1}\left(\frac{P_0}{2 \times (r + R_0)}\right)$ and, R_1 and R_2 are inner and outer radius of any ring,

2.3.3. Bubble model

Modeling the coolant density variation in the form of bubbles needs very unique perspective as sizes and positions of the bubbles inside the coolant is random due to nature of boiling process. In the bubble model, bubbles individually exist inside the coolant by representing a volume of void (superheated water) and have their own thermodynamic properties. To overcome such a problem, Monte Carlo method is implemented to randomly determine bubble's locations, sizes and shapes, and also to calculate occupied volume by non-spherical bubbles.

To estimate the bubble's locations, the distance from the center of the fuel rod to the edge of unit cell, denoted by ℓ and labeled in Fig. 1, is firstly estimated by Eq. (8)

$$\ell = \left[(\sqrt{2} - 1) \times \xi + 1 \right] \times \left(\frac{P_0}{2} + r_b \right) \quad (8)$$

where ξ is a random real number varying between 0 and 1. ℓ is the function of θ varying from 0 to $\pi/4$. Inserting Eq. (8) into Eq. (9) yields the angle between x-axis and position of the bubble.

$$\mu = \cos \theta = \left(\frac{P_0}{2} + r_b \right) / \ell \quad (9)$$

Note that a bubble radius is added to the above equations to form non-spherical bubbles. The radial void distribution is expressed in Eq. (3) for a cylindrical hollow tube. It needs to be reformulate for a cylindrical annular tube, more like an equivalent cylindrical unit cell where a fuel rod is located at the center. The distribution function away from the fuel rod surface becomes as follows

$$F(r) = \left(\frac{r - R_0}{\ell - R_0} \right)^n \quad (10)$$

For $R_0 \leq r \leq \ell$. As can be seen, the distribution treats as cumulative distribution function by ranging between 0 and 1. Then, applying direct samplings yields

$$\left(\frac{r - R_0}{\ell - R_0} \right)^n = \eta \quad (11)$$

where η is a random real number ranging from 0 to 1. Thus,

$$r = R_0 + (\ell - R_0) \times \eta^{1/n} \quad (12)$$

Finally, x and y coordinates of the center of a bubble with radius r_b is obtained from

$$\begin{aligned} x &= r \times \mu \times (-1)^\beta \\ y &= r \times \sqrt{1 - \mu^2} \times (-1)^\zeta \end{aligned} \quad (13)$$

where n , the exponential power given in the void distribution function, is a real number between 0 and ∞ . ζ and β are integer random numbers, as 0 or 1, and used to position the bubble in any quadrant of the coordinate system. Position of the bubble on the z-axis is simply calculated from Eq. (14)

$$z = (2\zeta - 1) \times \left(\frac{h}{2} + r_b \right) \quad (14)$$

where ζ is a random real number ranging from 0 to 1. The height of the unit cell, h , is assumed to be 1 cm. Using the definitions of

bubble volume (V_b) and the associated void fraction (α_b), void fraction occupied by a full bubble in the coolant with V_c volume becomes:

$$\alpha_b = \frac{V_b}{V_c} = \frac{4\pi r_b^3 / 3}{(P_0^2 - \pi R_0^2) \times h} \quad (15)$$

It is assumed that no bubbles touch each other. On the other hand, not all bubbles have a full sphere-shaped as they intersect with the edges of the unit cell and/or the fuel rod surface. Volumes of these bubbles (like Viviani's curve or partial sphere) are precisely calculated using hit-or-miss Monte Carlo technique with a negligible computational error of 0.0001 cm³ from the real value; therefore, bubbles with a volume of less than 0.0001 cm³ is ignored in modeling and is re-modelled. The volumes are enclosed by the following equations: the sphere surface with r_b radius and/or cylindrical surface of a fuel rod and/or unit cell surfaces.

$$\begin{cases} (x - x_b)^2 + (y - y_b)^2 + (z - z_b)^2 = r_b^2 \\ (x - x_0)^2 + (y - y_0)^2 = R_0^2 \\ -P_0/2 \leq x \leq P_0/2; -P_0/2 \leq y \leq P_0/2; -h/2 \leq z \leq h/2 \end{cases} \quad (16)$$

As a result, total void fraction occupied by the entire bubbles with different size and shape becomes

$$\alpha = \sum_i \alpha_b(i) \quad (17)$$

2.4. Computational procedure for bubble model

In the analyses, bubbles were randomly formed inside the sub-channel geometry by using Eqs. (8)–(14). Void fraction of each produced bubble was calculated by Eq. (15) after the analytical or Monte Carlo volume calculation. The temperature and density of the bubble were obtained from Eq. (6) by using the relevant values given in Table 1. The total void fraction of the sub-channel was estimated from Eq. (17). Number of bubbles (n_b) was increased until the total void fraction reached a maximum void fraction (α_0) and then the sub-channel model was finalized. Subsequently, an (MCNP) input file was prepared using the information of the position, temperature and density of each bubble. Upon MCNP run, the tallies such as k_{inf} and reaction rates were extracted from the output file. An illustration of MCNP model formed by the aforementioned technique is given in Fig. 4.

Although an average bubble diameter obtained from Eq. (5) was used in the study, effects of different void fraction, radial void distribution and bubble diameter on the reactor parameters were also examined. In addition to the fixed bubble radius, a randomly varying bubble diameter was studied in order to estimate the range of deviation from the homogeneous model. The studied parameters are shown in Table 2. $n = 1$ means that bubbles can settle into anywhere with equal probability.

It should be noted here that a single sub-channel model is regarded as an instant view of the unit cell. In fact, bubbles in a real sub-channel move constantly in the direction of r , θ and z depending on the time. This motion is defined in void fraction as a time-averaged value given in Eq. (2). Unluckily, there is no way to simulate a moving system in MCNP. Instead, to obtain the time-averaged void fraction, a set of sub-channel models (n_s) were generated until an average of k_{inf} of the models converged. Average k_{inf} is expressed in the form of

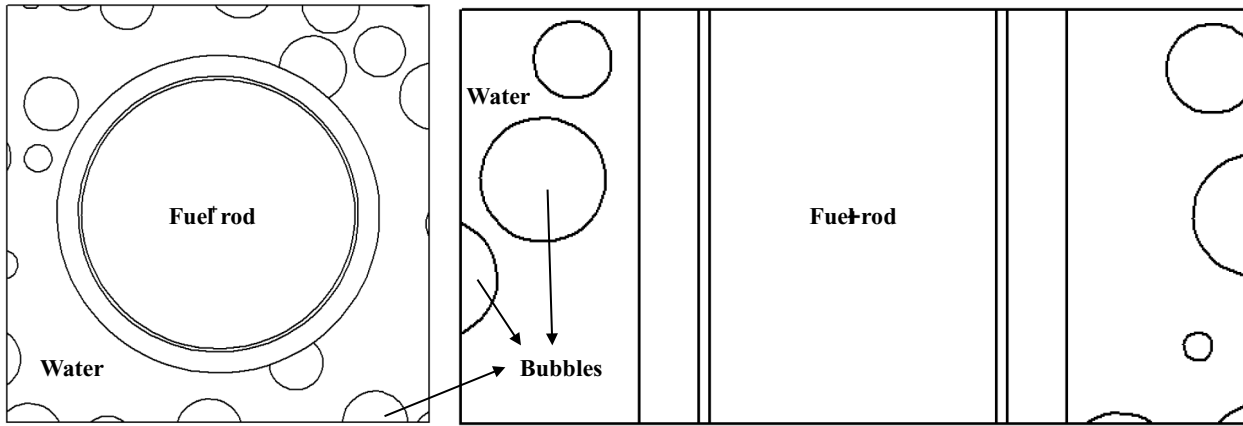


Fig. 4. Bubble model in MCNP: radial (left) and axial (right) views.

Table 2

Parameters studied in the bubble model.

Parameter	Unit	Value
Flow type		Bubbly
Bubble diameter (d_b)	mm	≤ 4
Void coefficient	%	≤ 25
Exponential power (n)		1–7

$$k_{inf}^{ave} = \frac{1}{n_s} \sum_{i=1}^{n_s} k_{inf}(i) \quad (18)$$

with a standard deviation of

$$\sigma = \sqrt{\frac{\frac{1}{n_s} \sum_{i=1}^{n_s} (k_{inf}(i) - k_{inf}^{ave})^2}{n_s - 1}} \quad (19)$$

The standard deviation is selected as a convergence criterion. The modeling stops when the deviation ($\delta\sigma$) in the standard deviation is less than 1 pcm. This value ensures that there would be no bias on the k_{inf}^{ave} value when compared with the computational error of about 10 pcm in k_{inf} . Results of bubble model were compared with the results of homogenous model. The computational flow chart for the bubble model is shown in Fig. 5.

Monte Carlo calculations for neutron transport were carried

out by using MCNP6.1.1 [18] code. For criticality analysis, up to 120000 source neutrons per cycle, 100 passive cycles and a total of 600 cycles were used. This thus yields the computational errors of k_{inf} in the homogenous, ring and bubble models are less than 2, 6 and 9 pcm, respectively and a relative standard deviation no more than 0.1% in the reaction rate tally cards. Temperature-dependent neutron cross-section libraries used in MCNP were produced using NJOY99 from the ENDF/B-VII.1 library. The $S(\alpha, \beta)$ approach was used for the interactions of neutrons in water and fuel regions. The “white boundary condition” was used at the surfaces of the unit cell since the periodic boundary condition was not applicable due to the fact that presence of the bubbles preclude the creation of a repetitive geometry. It is important to note that, the smallest bubble radius in modeling needs to be limited since the computational error in the reaction rate tallies of bubbles as well as computing time increases as the bubble radius decreases. Therefore, for the aforementioned neutron history, minimum bubble radius was obtained to be 0.5 mm yielding no more than 10% error, the recommended value in MCNP for reliable results.

3. Results and discussion

3.1. Homogenous model

Change of excess reactivity with void fraction is plotted in Fig. 6. Void fraction is changed from 0 to 25% by inserting into Eq. (1). The

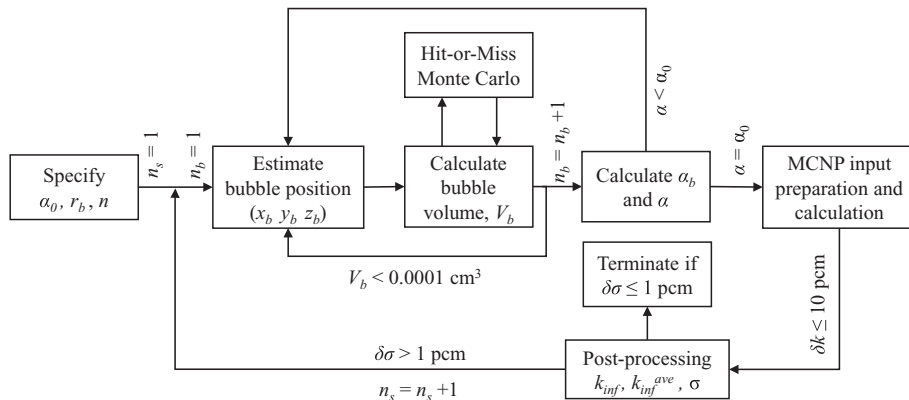


Fig. 5. Computational flow chart for the bubble model.

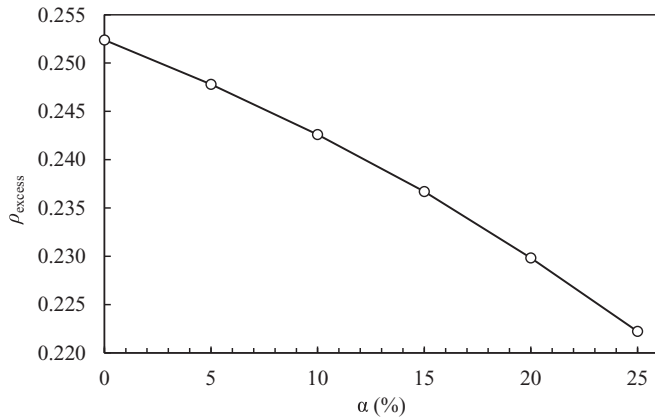


Fig. 6. Effect of void fraction on excess reactivity.

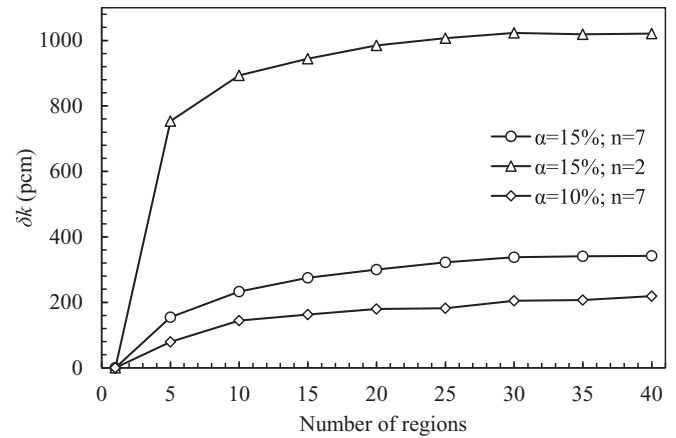


Fig. 8. Change of δk with number of regions.

relationship between reactivity and void fraction seems quasi-linear as in the relationship between the void fraction and the density. It can be expressed in the form of second order polynomial with an R squared value of 1 as follows

$$\rho_{\text{excess}}(\alpha) = -1.551 \times 10^{-5} \times \alpha^2 + 8.161 \times 10^{-4} \times \alpha + 0.2535 \quad (20)$$

where α is the void fraction (%). Excess reactivity decreases with increasing void fraction due to negative void feedback (reducing thermalization). Normalized thermal and fast flux distributions are illustrated in Fig. 7. The distribution has a one-eight symmetry. The highest thermal flux was calculated at the corners of the unit cell. Fast flux is simply opposite of the thermal flux distribution.

3.2. Ring model

For different values of α and n , variation of δk (from the homogenous model) by number of regions is displayed in Fig. 8. It is calculated that the difference increases with increasing number of regions and approaches a maximum value after about 30 regions. At the same α , an increase in n results in a decrease in δk , while at the same n , an increase in α results in an increase in δk . For the $n = 7$, the highest difference is found to be about 720 ± 6 pcm at

$\alpha = 25\%$. For the cases of number of region of 30 and $n = 7$, deviations (as percentage change) in two-group flux distributions are displayed in Fig. 9. The results indicate that thermal flux values of the ring model are higher than that of homogenous case throughout the unit cell, specifically up to 0.8% in the form of a circular belt around the fuel rod. In contrast to thermal flux, a lower fast flux distribution of up to 1.7% was obtained. This difference falls by 1.3% in the fuel region. It is understood that the representing the coolant in the form of rings causes a net effect on the reactor parameters. This effect may exceed 1000 pcm depending on the used void profile and fraction.

3.3. Bubble model

Average k_{inf} and its corresponding standard deviation vary with number of cycles. A sample case selected as $\alpha = 15\%$ and $n = 2$ is illustrated in Fig. 10. As seen from the graph, $k_{\text{inf}}^{\text{ave}}$, similarly its standard deviation, converges and becomes stable after about half of total cycles. From the results, the $k_{\text{inf}}^{\text{ave}}$ at about 100th cycles was found to be 1.30977 with a standard deviation of 23 pcm. In addition, it was observed that the distribution of individual k_{inf} values fits a normal distribution. The distribution obtained for the sample case is given in Fig. 11. It should be noted that there are also similar behaviors for all other cases.

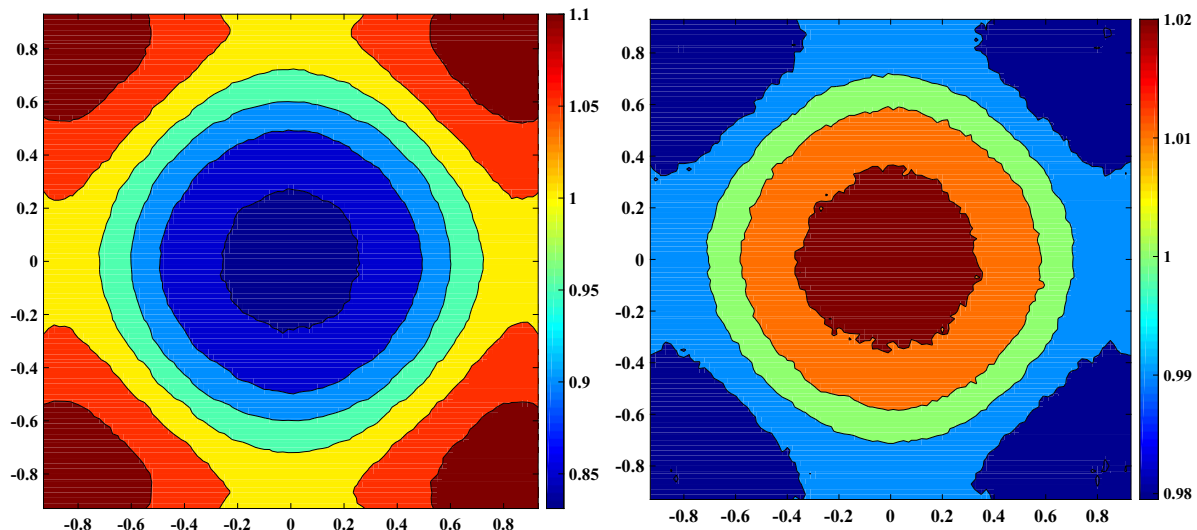


Fig. 7. Normalized flux distribution: thermal (left) and fast flux (right).

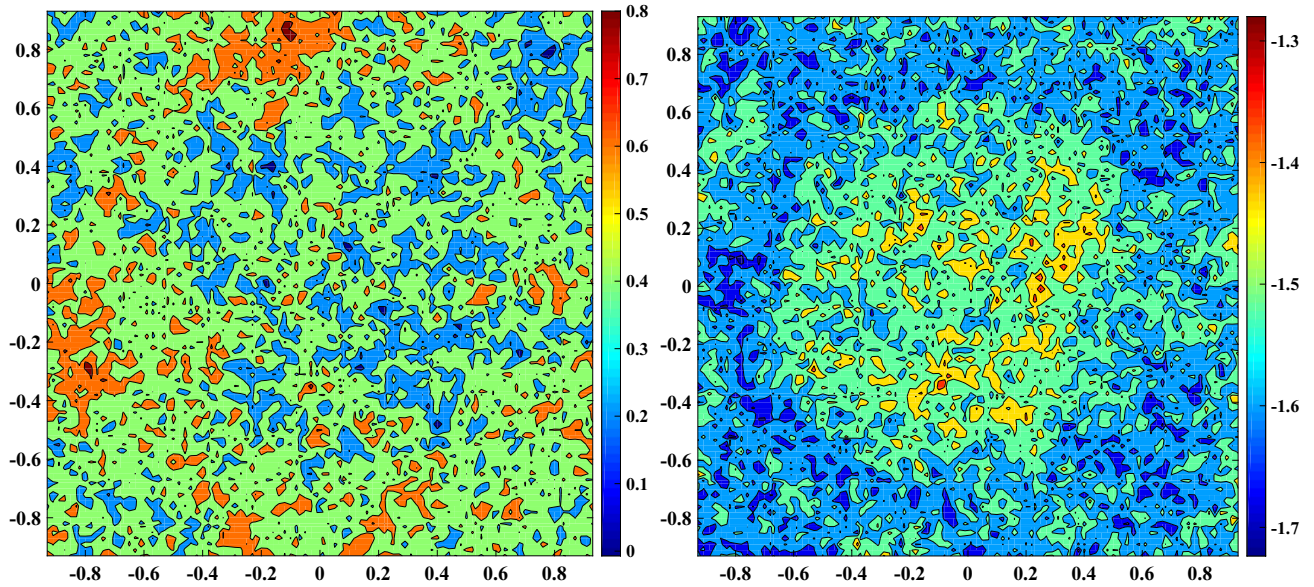


Fig. 9. Deviation (%) from the homogenous model: thermal (left) and fast flux (right).

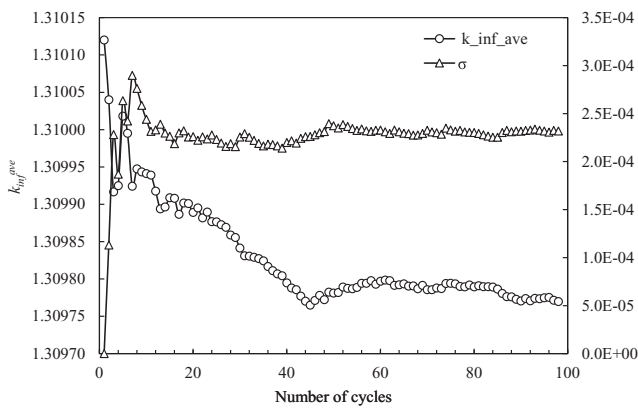


Fig. 10. Change of k_{inf}^{ave} with number of cycles.

For various void fraction and radial void profile, the converged k_{inf}^{ave} values with corresponding standard deviations and δk [$= k_{inf}^{ave} - k^{hom}$] are listed in Table 3. As first noticed from the table, for a selected void fraction, σ value of the k_{inf}^{ave} is inversely related to the void profile. Standard deviation, and also k_{inf}^{ave} , decreases as exponential power increases. In other words, k_{inf}^{ave} less fluctuates. It can be concluded that as the bubbles form near and/or on unit cell surfaces, neutron thermalization and thus the k_{inf}^{ave} reduces, even in the same void fraction. Spatial effects become important. It is also understood that the standard deviation depends on void fraction. For a selected exponential power, k_{inf}^{ave} value decreases as the void fraction increases. This is consistent with the results presented in the homogeneous model due to reducing coolant density. Lastly, standard deviation and k_{inf}^{ave} increase as the bubble diameter increases. As the bubble radius decreases, the deviation from the homogenous model increases.

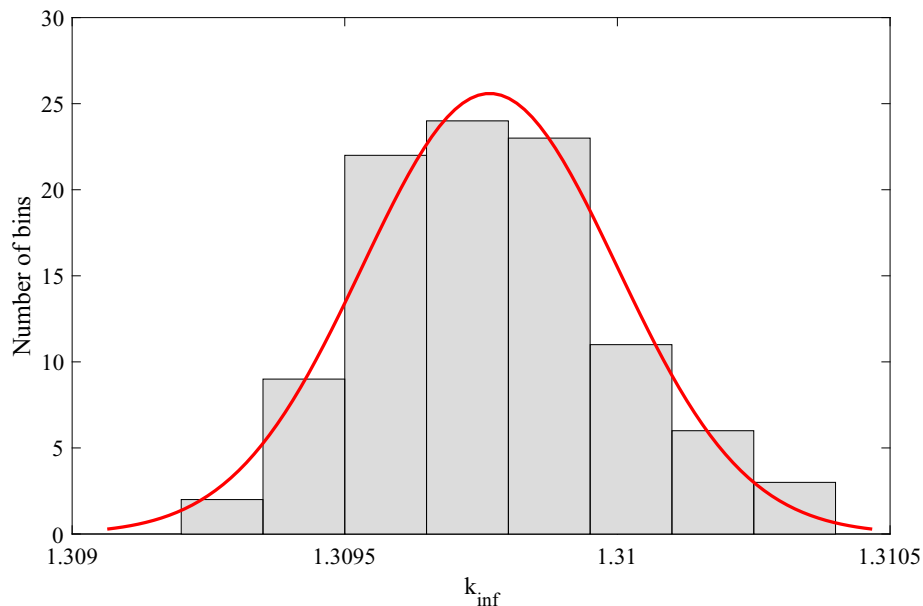


Fig. 11. Distribution of k_{inf} values.

Table 3
Converged k_{inf}^{ave} values for various void fractions and profiles.

α (%)	r_b (cm)	n	k_{inf}^{ave}	σ^a ($\pm$ pcm)	δk^b [= $k_{inf}^{ave} - k^{hom}$] (pcm)
5	0.15	1	1.32943	15	0
		2	1.32932	17	-11
		4	1.32925	15	-18
		7	1.32920	12	-23
			1.32943		
10	homogenous 0.15	1	1.32027	19	-3
		2	1.32007	19	-23
		4	1.31995	18	-35
		7	1.31992	15	-38
			1.32030		
15	homogenous 0.15	1	1.31008	23	-4
		2	1.30977	23	-35
		4	1.30962	20	-50
		7	1.30948	14	-64
			1.31012		
20	homogenous 0.15	1	1.29871	24	30
		2	1.29842	25	1
		4	1.29817	20	-24
		7	1.29798	20	-43
			1.29841		
25	homogenous 0.15	1	1.28621	24	49
		2	1.28587	23	15
		4	1.28559	23	-13
		7	1.28535	24	-37
			1.28572		
15	homogenous 0.05 0.10 0.15 0.20 0.05–0.20 (random)	2	1.30949	11	-63
			1.30962	14	-50
			1.30977	23	-35
			1.30986	32	-26
			1.30979	22	-33

^a This value is calculated from Eq. (19).

^b Computational error of δk is about ± 9 pcm.

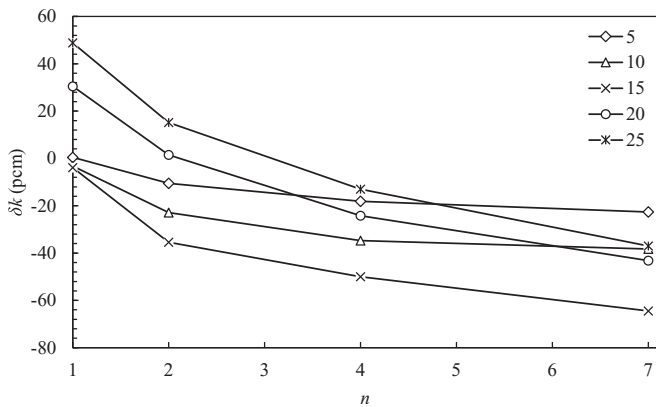


Fig. 12. Change of δk with void profile.

The smaller bubble, the less thermalization. In the case of further reduction in the bubble radius, that is, radius goes to zero, the bubble model may be expected to approach the homogenous model, $\delta k \rightarrow 0$. However, the opposite is the case. This situation may be attributed to the dynamics of Monte Carlo method used in modeling bubbles. Considering the bubbles placed according to the distribution function and the non-contact conditions, the smaller bubbles accumulate locally and quantitatively more in places closer to the unit cell surface than the larger bubbles. This negatively affects the thermalization, thus reducing the k_{inf} value. For the average bubble radius ($r_b = 1.5$ mm) used in this study, δk is found to be about 35 ± 9 pcm but 65 ± 9 pcm for $n = 7$. These results imply that bubble radius and void profile has a significant effect on the interaction of neutrons. Random diameter with

equal probability (lowermost row) gives the same results with the results of $\alpha = 15\%$ and $r_b = 0.15$ cm.

For various void fractions, variation of δk with exponential power is illustrated in Fig. 12. As the exponential power increases, that is, the bubbles pile up mostly around the unit cell periphery, deviation from the homogeneous model increases. In addition, the void fraction has an effect on the δk . The highest difference is observed at a void fraction of 15%. For less than 15% void fraction, $n = 1$ causes the lowest difference, while $n = 2$ for higher than 15% void fraction. For a selected exponential power, the results show that deviation from the homogenous model decreases up to 15% and then increases sharply. The main reason may be explained by the fact that bubbly flow shifts towards slug flow after 15% or a transition flow occurs. In spite of same decreasing trend of the deviation in δk , the bubbles are more tightly packed toward the unit cell surface when the void fraction is getting higher. Therefore, the void distributions used in this study may not be appropriate for the transition. As the bubble sizes become smaller, the bubbles become transparent to the neutrons, which can be thought of as another reason. However, in this case, the k_{inf}^{ave} value should have increased, not decreased, as the neutrons will exist more in the denser medium.

As shown in Fig. 13, the bubble model, obtained for $n = 7$, prominently deviates from the homogenous model. Deviations in flux distributions is more pronounced in certain regions, such as corners. The difference in thermal flux reach up to -1.5% at the corners whereas up to 0.6% at the center of the unit cell surfaces. The reason for the reduction at the corners is the accumulation of more bubbles in this region. Therefore, a less dense area constitutes, thus causing less thermalization. Although the difference in fast flux is not as high as in thermal flux, it increases at the locations where the thermal flux decreases, and increases at the locations where it decreases. Maximum difference is observed to be about -0.25% at the corners and center of unit cell surfaces.

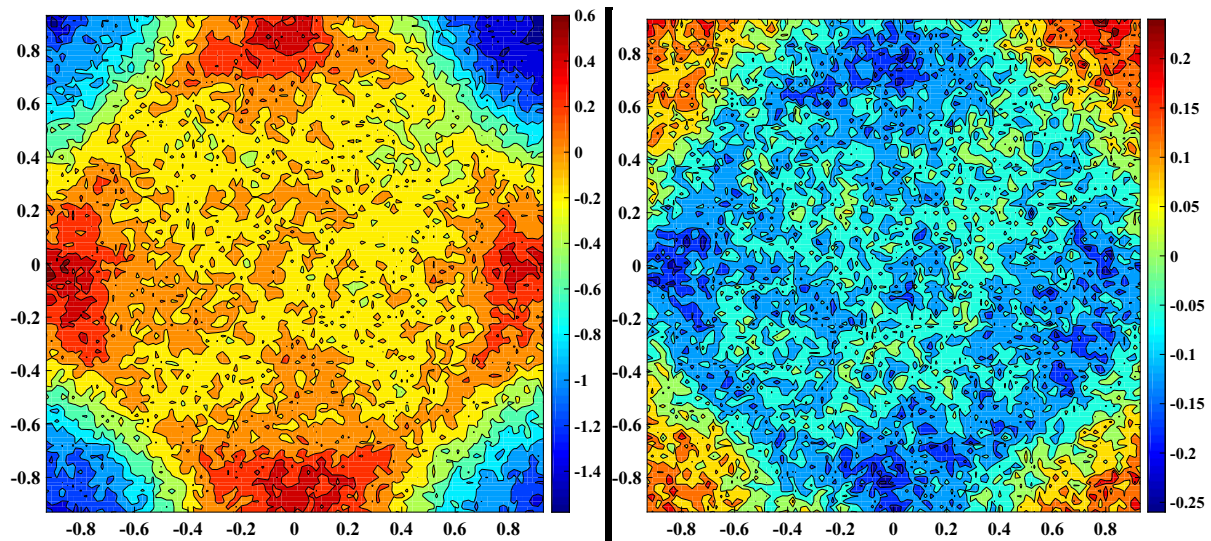


Fig. 13. Deviation (%) from the homogenous model: thermal (left) and fast flux (right).

4. Conclusion

The homogeneous model show that excess reactivity has a linear relation with void fraction. The results of the ring model indicate that coolant region must be divided into at least 30 rings. This number can be reduced by using the gradient-based method. In the ring model, the deviation in δk from the homogenous model increases as void fraction increases, and decreases as n increases. For the fully-developed bubbly flow, k_{inf} is overestimated by about 720 ± 6 pcm with respect to homogeneous model. Furthermore, the thermal flux at any point on the unit cell is higher while the fast flux is lower.

In the bubble model, the results show that the converged k_{inf}^{ave} values and its corresponding standard deviations strongly depend on the bubble size, bubble position, void fraction and void profile. Standard deviation reduces as exponential power increase and increases as the void fraction increases. Deviation in δk increases with increasing exponential power and decreases with increasing bubble radius. On the other hand, as the void fraction increases, the deviation increases until 15% and then decreases swiftly due to the beginning of a bubbly-to-slug flow transition after 15%. It appears that there is a strong but sophisticated relation between the flow properties and the distribution of k_{inf} . However, this relationship has a normal distribution. For the fully-developed bubbly flow at a void fraction of 15%, k_{inf}^{ave} and its standard deviation is calculated to be about -65 ± 9 and 15 pcm, respectively.

When the void fraction is modelled as bubbles, an average k_{inf} value that fluctuates resulting from the motion of the bubbles is found, instead of a single k_{inf} value obtained in homogeneous and ring models. This indicates that there is an oscillation in the criticality multiplication factor of the reactor in a real fuel channel. The magnitude of this oscillation differs by the flow regime type.

Bubbly flow was investigated in this study since it is relatively simple to model. However, it should be noted that assorted flow regimes are observed throughout the channel. As an example, for the slug flow, shape of the bubbles shifts from the ideal spherical form to a cylindrical form with semi-sphere cups at the tips. In addition to the shape, sizes of bubbles are quite unlike compared to the bubbly flow as well as positioning inside the sub-channel. Due to these reasons, the local effects would be significant in neutronic calculations. In a follow-up study, the effects of other flow regimes on the reactor parameters are to be examined. The following phase

of this study is going to be carried out in a full-length fuel channel to observe the combined effects of different flow types. The change in reactor parameters at 15% void fraction in bubbly flow is quite interesting. It is obvious that this change, indicating a transition flow, requires a more detailed investigation.

In this study, bubbles were simulated in the form of perfect spherical structure. Experimental studies, however, show that bubbles slightly differ from the spherical shape. It is clear that this difference will cause somewhat influence on the k_{inf} values calculated in the section of the bubble model. However, the magnitude of this effect cannot be known at this stage since the literally modeling of such a structure is nearly impractical.

To sum up, it is understood that the bubble model gives closer results to the homogeneous model whereas the ring model gives more distant results. It should be noted that, this does not imply that the homogeneous model is the correct approach. It is found that spatial effects are important in local values such as flux distribution even if they are not seen in the global values such as k_{inf} . Compared to the homogenous model, neutrons are less thermalized (under-moderated) in the bubble model, while more thermalized (over-moderated) in the ring model. In terms of the interaction of neutrons, it is observed that there are notable differences between neutron transport in a medium with a uniform density and neutron transport in a layered medium the density of which gradually decreases, or neutron transport in a medium containing local sharp density changes at short distances. As a result, the bubble model in modeling the void fraction can be considered as an important step and used for accurate and precise neutronic, thermal-hydraulic and safety calculations in nuclear engineering.

Conflicts of interest

The authors declare no conflict of interest.

References

- [1] S. Hosakawa, K. Hyashi, A. Tomiyama, Void distribution and bubble motion in bubbly flows in a 4×4 rod bundle. Part I: Experiments, *J. Nucl. Sci. Technol.* 51 (2) (2014) 220–230.
- [2] A. Tomiyama, Y. Nakahara, Y. Adachi, S. Hosokawa, Shapes and rising velocities of single bubbles rising through an inner subchannel, *J. Nucl. Sci. Technol.* 40 (3) (2003) 136–142.

- [3] R.H.S. Winterton, J.S. Munaweera, Bubble size in two-phase gas–liquid bubbly flow in ducts, *Chem. Eng. Process* 40 (2001) 437–447.
- [4] T. Hibiki, R. Situ, Y. Mi, M. Ishii, Modelling of bubble-layer thickness for formulation of one-dimensional interfacial area transport equation in subcooled boiling two-phase flow, *Int. J. Heat Mass Transf.* 46 (2003) 1409–1423.
- [5] J.E. Julia, T. Hibiki, M. Ishii, B.-J. Yun, G.-C. Park, Drift-flux model in a sub-channel of rod bundle geometry, *Int. J. Heat Mass Transf.* 52 (2009) 3032–3041.
- [6] O. Marfaing, M. Guingo, J. Lavieville, G. Bois, N. Mechtoua, N. Merigoux, S. Mimouni, An analytical relation for the void fraction distribution in a fully developed bubbly flow in a vertical pipe, *Chem. Eng. Sci.* 152 (2016) 579–585.
- [7] T. Ikehara, Y. Kudo, M. Tamitani, M. Yamamoto, Effect of subchannel void fraction distribution on lattice physics parameters for boiling water reactor fuel bundles, *J. Nucl. Sci. Technol.* 45 (12) (2008) 1237–1251.
- [8] T. Ama, H. Hyoudou, T. Takeda, Effect of radial void distribution within fuel assembly on assembly neutronic characteristics, *J. Nucl. Sci. Technol.* 39 (1) (2002) 90–100.
- [9] A. Bennett, N. Martin, M. Avramova, K. Ivanov, Impact of radial void fraction distribution on boiling water reactor lattice physics calculations: application to AREVA's next generation BWR fuel assembly, the ATRIUM™ 11 design, in: *PHYSOR 2016*, Sun Valley, Idaho, May 1–5, 2016.
- [10] F. Jatuff, F.D. Giust, J. Krouthén, S. Helmersson, R. Chawla, Effects of void uncertainties on the void reactivity coefficient and pin power distributions for a 10x10 BWR assembly, *Ann. Nucl. Energy* 33 (2006) 119–125.
- [11] NEA, NUPEC BWR Full-Size Fine-mesh Bundle Test (BFBT) Benchmark: Volume 1-Specifications, vol. 5, NEA/NSC/DOC, 2005. OECD/NEA, 2006.
- [12] NEA, Benchmark for Uncertainty Analysis in Modelling (UAM) for the Design, Operation and Safety Analysis of LWRs: Specification and Support Data for Neutronics Cases (Phase I), vol. 1, NEA/NSC/DOC, 2013. OECD/NEA, 2013.
- [13] NEA, Boiling Water Reactor Turbine Trip (TT) Benchmark, Volume I: Final Specifications, vol. 1, NEA/NSC/DOC, 2001. OECD/NEA, 2001.
- [14] M. Ishii, T. Hibiki, *Thermo-Fluid Dynamics of Two-phase Flow*, second ed., Springer, New York, 2011.
- [15] S. Levy, *Two-Phase Flow in Complex Systems*, John Wiley & Sons, New York, 1999.
- [16] N.E. Todreas, M.S. Kazimi, *Nuclear Systems I: Thermal Hydraulic Fundamentals*, second ed., CRC Press, Taylor & Francis Group, USA, 2012.
- [17] R. Luo, Q. Song, X. Yang, Z. Wang, Developed 'laminar' bubbly flow with non-uniform bubble sizes, *Sci. China, Ser. E* 44 (1) (2001) 47–54.
- [18] D.B. Pelowitz, A.J. Fallgren, G.E. McMath, *MCNP6 User's Manual*, Code Version 6.1.1beta, Manual Rev. 0, LA-CP-14-00745, Rev. 0, 2014.