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σ-core and $\mathcal{I}$-core of bounded sequences

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Abstract

In this paper we characterize matrices that map every bounded sequence into one whose σ-core is a subset of the $\mathcal{I}$-core of the original sequence.
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Keywords: $\mathcal{I}$-convergent sequence; $\mathcal{I}$-limit superior and inferior; σ-core of a sequence

1. Introduction

If K is a subset of the natural numbers $\mathbb{N}$, K_n will denote the set $\{k \in K\colon k \leqslant n\}$ and $|K_n|$ will denote the cardinality of K_n. The natural density of K [25] is given by $\delta(K) := \lim_n (1/n)|K_n|$, if it exists. Fast introduced the definition of statistical convergence using the natural density of a set. The number sequence $x = (x_k)$ is statistically convergent to L provided that for every $\varepsilon > 0$ the set $K := K(\varepsilon) := \{k \in \mathbb{N}\colon |x_k - L| \geqslant \varepsilon\}$ has natural density zero [11,13]. Hence x is statistically convergent to L iff $(C_1\chi_{K(\varepsilon)})_n \to 0$ (as $n \to \infty$, for every $\varepsilon > 0$), where C_1 is the Cesáro mean of order one and χ_K is the characteristic function of the set K. Properties of statistically convergent sequences have been studied in [2,3,13,23,29].

Statistical convergence can be generalized by using a nonnegative regular summability matrix A in place of C_1.

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Following Freedman and Sember [12], we say that a set $K \subseteq \mathbb{N}$ has A-density if $\delta_A(K) := \lim_n (A\chi_K)_n = \lim_n \sum_{k\in K} a_{nk}$ exists, where $A = (a_{nk})$ is a nonnegative regular matrix.

The number sequence $x = (x_k)$ is A-statistically convergent to L provided that for every $\varepsilon > 0$ the set $K(\varepsilon)$ has A-density zero [3,12,23].

Fridy [14] has introduced the notions of statistical limit point and cluster point and then Fridy and Orhan [15] studied the idea of statistical limit superior and inferior. Connor and Kline [5] and Demirci [9] extended these concepts to A-statistical convergence using a nonnegative regular summability matrix A in place of C_1. Also Connor has introduced μ-statistical analogue of these concepts using a finitely additive set function μ taking values in $[0, 1]$ defined on a field Γ of subsets of $\mathbb{N}$ such that if $|A| < \infty$, then $\mu(A) = 0$; if $A \subset B$ and $\mu(B) = 0$, then $\mu(A) = 0$; and $\mu(\mathbb{N}) = 1$ [4,5].

The number sequence $x = (x_k)$ is μ-statistically convergent to L provided that $\mu(\{k \in \mathbb{N}\colon |x_k - L| \geqslant \varepsilon\}) = 0$ for every $\varepsilon > 0$ [4,5].

Let $X \neq \emptyset$. A class $S \subseteq 2^X$ of subsets of X is said to be an ideal in X provided that S is additive and hereditary, i.e., if S satisfies these conditions: (i) $\emptyset \in S$, (ii) $A, B \in S \Rightarrow A \cup B \in S$, (iii) $A \in S,\ B \subseteq A \Rightarrow B \in S$ [18]. An ideal is called nontrivial if $X \notin S$. A nontrivial ideal S in X is called admissible if $\{x\} \in S$ for each $x \in X$ [18].

Kostyrko et al. [18,19] introduced two types of "ideal convergence."

Let $\mathcal{I}$ be a nontrivial ideal in $\mathbb{N}$. A sequence $x = (x_k)$ of real numbers is said to be $\mathcal{I}$-convergent to L if for every $\varepsilon > 0$ the set $A(\varepsilon) := \{k \in \mathbb{N}\colon |x_k - L| \geqslant \varepsilon\}$ belongs to $\mathcal{I}$ [18]. In this case we write $\mathcal{I}\text{-}\lim x = L$.

Let $\mathcal{I}$ be an admissible ideal in $\mathbb{N}$. A sequence $x = (x_k)$ of real numbers is said to be $\mathcal{I}^*$-convergent to L if there is a set $H \in \mathcal{I}$, such that for $M = \mathbb{N}\backslash H = \{m_1 < m_2 < \cdots\}$ we have $\lim_k x_{m_k} = L$. In this case we write $\mathcal{I}^*\text{-}\lim x = L$ [18].

For every admissible ideal $\mathcal{I}$ the following relation between them holds: Let $\mathcal{I}$ be an admissible ideal in $\mathbb{N}$. If $\mathcal{I}^*\text{-}\lim x = L$ then $\mathcal{I}\text{-}\lim x = L$ [18].

Note that for some ideals the converse of this result holds (see [18, Example 3.1]). Kostyrko et al. have given the necessary and sufficient condition for equivalence of $\mathcal{I}$ and $\mathcal{I}^*$-convergences. This condition is similar to the additive property for null sets in [4,12].

An admissible ideal $\mathcal{I}$ in $\mathbb{N}$ is said to satisfy the additive property if for every countable system $\{A_1, A_2, \ldots\}$ of mutually disjoint sets belonging to $\mathcal{I}$ there exist sets $B_j \subseteq \mathbb{N}$ ($j = 1, 2, \ldots$) such that the symmetric differences $A_j \triangle B_j$ ($j = 1, 2, \ldots$) are finite and $B = \bigcup_{j=1}^{\infty} B_j$ belong to $\mathcal{I}$ [18].

It is known [18] that $\mathcal{I}\text{-}\lim x = L \Leftrightarrow \mathcal{I}^*\text{-}\lim x = L$ if and only if $\mathcal{I}$ has the additive property [18]. Some results on $\mathcal{I}$-convergence may be found in [10,18,19].

Demirci [10] has introduced the concepts of $\mathcal{I}$-limit superior and inferior and studied some properties of $\mathcal{I}$-limit superior and inferior for a sequence of real numbers.

For a real number sequence $x = (x_k)$ let B_x denote the set

$$B_x := \big\{b \in \mathbb{R}\colon \{k\colon x_k > b\} \notin \mathcal{I}\big\}.$$

Similarly,

$$A_x := \big\{a \in \mathbb{R}\colon \{k\colon x_k < a\} \notin \mathcal{I}\big\}.$$

Let $\mathcal{I}$ be an admissible ideal and x be a real number sequence. Then the $\mathcal{I}$-limit superior of x is given by

$$\mathcal{I}\text{-}\limsup x := \begin{cases} \sup B_x & \text{if } B_x \neq \phi, \\ -\infty & \text{if } B_x = \phi. \end{cases}$$

Also, the $\mathcal{I}$-limit inferior of x is given by

$$\mathcal{I}\text{-}\liminf x := \begin{cases} \inf A_x & \text{if } A_x \neq \phi, \\ +\infty & \text{if } A_x = \phi. \end{cases}$$

Recall [10] that for a real number sequence $x = (x_k)$ the number β is the $\mathcal{I}\text{-}\limsup x$ if and only if for every $\varepsilon > 0$,

$$\{k\colon\ x_k > \beta - \varepsilon\} \notin \mathcal{I} \quad \text{and} \quad \{k\colon\ x_k > \beta + \varepsilon\} \in \mathcal{I}.$$

Also the number α is the $\mathcal{I}\text{-}\liminf x$ if and only if for every $\varepsilon > 0$,

$$\{k\colon\ x_k < \alpha + \varepsilon\} \notin \mathcal{I} \quad \text{and} \quad \{k\colon\ x_k < \alpha - \varepsilon\} \in \mathcal{I}.$$

The $\mathcal{I}$-bounded sequence x is $\mathcal{I}$-convergent if and only if $\mathcal{I}\text{-}\limsup x = \mathcal{I}\text{-}\liminf x$ [10].

Note that if we define $\mathcal{I}_{\delta_A} = \{K \subseteq \mathbb{N}\colon\ \delta_A(K) = 0\}$, $\mathcal{I}_{\delta_{C_1}} = \{K \subseteq \mathbb{N}\colon\ \delta(K) = 0\}$ and $\mathcal{I}_\mu = \{K \in \Gamma\colon\ \mu(K) = 0\}$, then we get Definition 1 of [9], Definition 1 of [15] and Connor's definitions [6] of μ-statistical superior and inferior, respectively.

Note that the sets $\mathcal{I}_{\delta_A}$, $\mathcal{I}_{\delta_{C_1}}$ and $\mathcal{I}_\mu$ have the additive property for null sets [6,12]. Throughout the paper $\mathcal{I}$ will have the additive property.

In this paper we characterize matrices that map every bounded sequence into one whose σ-core is a subset of the $\mathcal{I}$-core of the original sequence. The final result follows a result of Choudhary [1] in giving conditions on matrices T and H so that the σ-core of Tx is contained in the $\mathcal{I}$-core of Hx.

2. σ-core and $\mathcal{I}$-core

Let ℓ^∞ and c be the space of real bounded and convergent sequences, respectively. Let σ be a mapping of the set of positive integers into itself. A continuous linear functional φ on ℓ^∞ is said to be an invariant mean or a σ-mean if and only if (1) $\varphi(x) \geqslant 0$ when the sequence $x = (x_n)$ has $x_n \geqslant 0$ for all n, (2) $\varphi(e) = 1$, where $e = (1, 1, 1, \ldots)$, and (3) $\varphi((\sigma_{(n)})) = \varphi(x)$ for all $x \in \ell^\infty$. We assume throughout this paper $\sigma^j(n) \neq n$ for all $n > 0$, $j \geqslant 1$. For certain kinds of mappings σ, every invariant mean φ extends the limit functional on the space c, in the sense that $\varphi(x) = \lim x$ for all $x \in c$. Consequently, $c \subset V_\sigma$, where V_σ is the set of bounded sequences all of whose σ-means are equal. It is known [30] that

$$V_\sigma = \Big\{x \in \ell^\infty\colon\ \lim_p t_{pn}(x) = s \text{ uniformly in } n,\ s = \sigma\text{-}\lim x\Big\},$$

where $p \geqslant 0$, $n > 0$, and

$$t_{pn}(x) = \frac{x_n + x_{\sigma(n)} + x_{\sigma^2(n)} + \cdots + x_{\sigma^p(n)}}{p+1}. \tag{1}$$

Let $V : \ell^\infty \to \mathbb{R}$ be defined by

$$V(x) = \limsup_p \sup_n t_{pn}(x),$$

where $t_{pn}(x)$ is given by (1). The σ-core of x is defined to be the closed interval

$$\sigma\text{-core}\{x\} := \big[-V(-x), V(x)\big] \quad [21].$$

If we take $\sigma(n) = (n+1)$ then σ-convergence and almost convergence coincide [20]. In this case σ-core$\{x\}$ is the same as $\mathcal{B}$-core$\{x\}$ [8,21,26].

Note that if x is an $\mathcal{I}$-bounded real sequence, then

$$\mathcal{I}\text{-core}\{x\} = [\mathcal{I}\text{-}\liminf x, \mathcal{I}\text{-}\limsup x] \quad [10].$$

In [15] Fridy and Orhan introduced the concept of the statistical core of a real number sequence, and proved the statistical core theorem. Those results have also been extended to the complex case by them too [16]. Using the same technique as in [16], the concept of $\mathcal{I}$-core of a complex sequence is introduced in [10], and necessary conditions are established for a summability matrix A to yield $\mathcal{I}$-core$\{Ax\} \subseteq \mathcal{I}$-core$\{x\}$ whenever x is a bounded complex number sequence.

We note that if we define $\mathcal{I} := \mathcal{I}_{C_1}$ and $\sigma(n) = (n+1)$, then $\mathcal{I}$-convergence and σ-convergence are incomparable (see [24]).

Let $T = (t_{nk})$ be an infinite matrix, and let X and Y be two sequence spaces. If Tx exists for each $x \in X$ and $Tx \in Y$ then we say that T maps X into Y. The set of matrices that map X into Y is denoted by (X, Y). The set of matrices that map X into Y and leave the limit or sum invariant is denoted by $(X, Y; p)$. For example, if $T \in (c, c; p)$, then $\lim Tx = \lim x$ for every $x \in c$. In this case T is called regular (see [7,28]).

Schaefer [30] has characterized the classes of matrices $(c, V_\sigma; p)$ and (ℓ^∞, V_σ).

By $F_\mathcal{I}$ and $F_\mathcal{I}(b)$ we denote the set of all the $\mathcal{I}$-convergent sequences and all $\mathcal{I}$-convergent bounded sequences.

Note that if $\mathcal{I}$ has the additive property, then using the idea given in Theorem 4.1 of [17] we can obtain necessary and sufficient conditions to characterize the matrices of the class $(F_\mathcal{I} \cap X, Y)$, where X is a section-closed sequence space containing e and Y an arbitrary sequence space. The proof is similar to Kolk's proof. So we omit the details.

Now if we take $X = \ell^\infty$ and $Y = V_\sigma$, then we get necessary and sufficient conditions to characterize the matrices of the class $(F_\mathcal{I}(b), V_\sigma; p)$.

Hence $T \in (F_\mathcal{I}(b), V_\sigma; p)$ if and only if $T \in (c, V_\sigma; p)$ and $T^{[K]} \in (\ell^\infty, V_\sigma)$ for every $K \in \mathcal{I}$, where $T^{[K]} = (d_{nk})$ is given by $d_{nk} = t_{nk}$ if $k \in K$; $K \in \mathcal{I}$ and $d_{nk} = 0$ otherwise.

For typographical convenience we shall use the notation $t(n,k)$ to denote the element t_{nk} of the matrix T.

By Theorems 1 and 2 of [30], this is equivalent to the following

Proposition 1. $T \in (F_\mathcal{I}(b), V_\sigma; p)$ *if and only if*

(i) $\displaystyle \sup_n \sum_{k=1}^{\infty} \big|t(n,k)\big| < \infty;$

(ii) $\displaystyle \sigma\text{-}\lim_n t(n,k) = 0$ *for every* k;

(iii) $\sigma\text{-}\lim_n \sum_{k=1}^{\infty} t(n,k) = 1$; *and*

(iv) $\lim_p \sum_{k \in K} \left| \frac{1}{p+1} \sum_{j=0}^{p} t\bigl(\sigma_{(n)}^{j}, k\bigr) \right| = 0$ *uniformly in n for every $K \in \mathcal{I}$.*

Now we have

Theorem 2. *Let $T : l^\infty \to l^\infty$ and $\gamma(x) := \mathcal{I}\text{-}\limsup x$. Then*

$$V(Tx) \leqslant \gamma(x) \quad \textit{for every } x \in l^\infty \tag{2}$$

if and only if

(a) $T \in \bigl(F_{\mathcal{I}}(b), V_\sigma; p\bigr)$;

(b) $\lim_p \sum_{k=1}^{\infty} \left| \frac{1}{p+1} \sum_{j=0}^{p} t\bigl(\sigma_{(n)}^{j}, k\bigr) \right| = 1$ *uniformly in n.*

Proof. Assume (2) holds and $x \in l^\infty$. Then $Tx \in l^\infty$; and also we have

$$-\gamma(-x) \leqslant -V(-Tx) \leqslant V(Tx) \leqslant \gamma(x).$$

If $x \in F_{\mathcal{I}}(b)$, then $\gamma(x) = -\gamma(-x)$, hence T maps $F_{\mathcal{I}}(b)$ into V_σ and $\mathrm{V}_\sigma\text{-}\lim Tx = \mathcal{I}\text{-}\lim x$, which proves (a).

Next we are going to show that

$$\lim_p \sum_{k=1}^{\infty} \bigl|t(p,n,k)\bigr| = 1 \quad \text{uniformly in } n,$$

where

$$t(p,n,k) := \frac{1}{p+1} \sum_{j=0}^{p} t\bigl(\sigma_{(n)}^{j}, k\bigr).$$

By condition (iii) of Proposition 1,

$$\liminf_p \sup_n \sum_{k=1}^{\infty} \bigl|t(p,n,k)\bigr| \geqslant \liminf_p \sum_{k=1}^{\infty} t(p,n,k) \to 1 \quad \text{as } p \to \infty.$$

Hence,

$$\liminf_p \sup_n \sum_{k=1}^{\infty} \bigl|t(p,n,k)\bigr| \geqslant 1.$$

By a theorem similar to Theorem 12 of [31] it is proved in [22] that there is a sequence $x \in \ell^\infty$ such that $\|x\|_\infty \leqslant 1$ and

$$\limsup_p \sup_n \sum_{k=1}^{\infty} t(p,n,k)x_k = \limsup_p \sup_n \sum_{k=1}^{\infty} \bigl|t(p,n,k)\bigr|. \tag{3}$$

Hence,

$$
\begin{aligned}
1 &\leqslant \liminf_p \sup_n \sum_{k=1}^{\infty} \bigl|t(p,n,k)\bigr| \leqslant \limsup_p \sup_n \sum_{k=1}^{\infty} \bigl|t(p,n,k)\bigr| \\
&= \limsup_p \sup_n \sum_{k=1}^{\infty} t(p,n,k)x_k \quad \text{(by (3))} \\
&\leqslant \gamma(x) \quad \text{(by hypothesis)} \\
&\leqslant \|x\|_\infty \leqslant 1
\end{aligned}
$$

from which we get (b).

Conversely assume (a) and (b) hold, and $x \in \ell^\infty$. Then $Tx \in \ell^\infty$ and $\gamma(x)$ is finite. Given $\varepsilon > 0$, let $E_1 := \{k\colon\ x_k > \gamma(x) + \varepsilon\}$. Hence $E_1 \in \mathcal{I}$ and if $k \in E_2 := \mathbb{N} \backslash E_1$ then $x_k \leqslant \gamma(x) + \varepsilon$. For any real number z we write $z^+ := \max\{z, 0\}$ and $z^- := \max\{-z, 0\}$, whence

$$
|z| = z^+ + z^-, \qquad z = z^+ - z^-, \qquad |z| - z = 2z^-.
$$

Let

$$
\begin{aligned}
t^+(p,n,k) &:= \frac{1}{p+1} \sum_{j=0}^{p} t^+\bigl(\sigma_{(n)}^j, k\bigr), \\
t^-(p,n,k) &:= \frac{1}{p+1} \sum_{j=0}^{p} t^-\bigl(\sigma_{(n)}^j, k\bigr).
\end{aligned}
$$

Then for a fixed positive integer m, we write

$$
\begin{aligned}
t_{pn}(Tx) &= \sum_{k<m} t(p,n,k)x_k + \sum_{k\geqslant m} t(p,n,k)x_k \\
&= \sum_{k<m} t(p,n,k)x_k + \sum_{k\geqslant m} t^+(p,n,k)x_k - \sum_{k\geqslant m} t^-(p,n,k)x_k \\
&= \sum_{k<m} t(p,n,k)x_k + \sum_{\substack{k\geqslant m\\ k\notin E_1}} t^+(p,n,k)x_k + \sum_{\substack{k\geqslant m\\ k\notin E_2}} t^+(p,n,k)x_k \\
&\quad - \sum_{k\geqslant m} t^-(p,n,k)x_k \\
&\leqslant \|x\|_\infty \sum_{k<m} \bigl|t(p,n,k)\bigr| + \bigl(\gamma(x)+\varepsilon\bigr) \sum_{k\geqslant m} \bigl|t(p,n,k)\bigr| \\
&\quad + \|x\|_\infty \sum_{k\geqslant m} \bigl|t(p,n,k)\bigr| + \|x\|_\infty \sum_{k\geqslant m} \bigl(\bigl|t(p,n,k)\bigr| - t(p,n,k)\bigr).
\end{aligned}
$$

Now Proposition 1 and (b) imply that

$$
V(Tx) \leqslant \gamma(x) + \varepsilon.
$$

Since ε is arbitrary we conclude that (2) holds hence the proof is completed. □

Similarly we could get $\alpha(x) := \mathcal{I}\text{-}\liminf x \leqslant -V(-Tx)$, and hence we have the following result.

Theorem 3. *If $T : l^\infty \to l^\infty$, then*

$$\sigma\text{-core}\{Tx\} \subseteq \mathcal{I}\text{-core}\{x\} \quad \textit{for every } x \in \ell^\infty$$

if and only if conditions (a) *and* (b) *of Theorem* 2 *hold.*

If we take $\sigma(n) = (n+1)$, then we get $V_\sigma = F$, the set of all almost convergent sequences. The case in which $I = I_{C_1}$ yields that $F_1(b) = \mathrm{st}(b)$, the set of all bounded statistically convergent sequences. Hence, the following two results are immediate from Theorem 3.

Corollary 4. *If $T : l^\infty \to l^\infty$, then*

$$\mathcal{B}\text{-core}\{Tx\} \subseteq \mathcal{I}\text{-core}\{x\} \quad \textit{for every } x \in \ell^\infty$$

if and only if

(i) $T \in (F_{\mathcal{I}}(b), F; p)$;

(ii) $$\lim_p \sum_{k=1}^{\infty} \left| \frac{1}{p+1} \sum_{j=0}^{p} t(n+j,k) \right| = 1 \quad \textit{uniformly in } n.$$

The next result appears as Theorem 4 in [27].

Corollary 5. *If $T : l^\infty \to l^\infty$, then*

$$\mathcal{B}\text{-core}\{Tx\} \subseteq \mathrm{st}\text{-core}\{x\} \quad \textit{for every } x \in \ell^\infty$$

if and only if

(i) $T \in \big(\mathrm{st}(b), F; p\big)$;

(ii) $$\lim_p \sum_{k=1}^{\infty} \left| \frac{1}{p+1} \sum_{j=0}^{p} t(n+j,k) \right| = 1 \quad \textit{uniformly in } n.$$

Theorem 6. *Let H be a triangular matrix with nonzero diagonal entries, and denote its triangular inverse by H^{-1}. For an arbitrary matrix T, in order that, whenever $Hx \in l^\infty$, Tx should exist and be bounded and satisfy*

$$\sigma\text{-core}\{Tx\} \subseteq \mathcal{I}\text{-core}\{Hx\}, \tag{4}$$

it is necessary and sufficient that

(i) $C := TH^{-1}$ *exists*;

(ii) $C \in \big(F_{\mathcal{I}}(b), V_\sigma; p\big)$;

(iii) $$\lim_p \sum_{k=1}^{\infty} \left| \frac{1}{p+1} \sum_{j=0}^{p} c\big(\sigma^j_{(n)}, k\big) \right| = 1;$$

$$\text{(iv)} \quad \lim_{\nu} \sum_{k=0}^{\nu} \left| \sum_{j=\nu+1}^{\infty} t(n,j)h^{-1}(j,k) \right| = 0 \quad \textit{for any fixed } n.$$

Proof. *Necessity.* If $(Tx)_n$ exists for each n whenever $Hx \in l^\infty$, then by Lemma 2 of Choudhary [1], (i) and (iv) hold. By the same lemma, we also have $Tx = Cy$, where $y = Hx$. By hypothesis $Tx \in l^\infty$, hence $Cy \in l^\infty$. Now (4) implies that σ-core$\{Cy\} \subseteq \mathcal{I}$-core$\{y\}$. By Theorem 3, we get (ii) and (iii).

Sufficiency. Conditions (i)–(iv) imply the conditions of Lemma 2 of Choudhary [1]; so, it follows from that lemma that $Cy \in l^\infty$, and hence $Tx \in l^\infty$. Now Theorem 3 yields that σ-core$\{Cy\} \subseteq \mathcal{I}$-core$\{y\}$, and since $y = Hx$ and $Cy = Tx$, we have σ-core$\{Tx\} \subseteq \mathcal{I}$-core$\{Hx\}$, whence the result follows. □

References

[1] B. Choudhary, An extension of Knopp's core theorem, J. Math. Anal. Appl. 132 (1988) 226–233.
[2] J. Connor, The statistical and strong p-Cesáro convergence of sequences, Analysis 8 (1988) 47–63.
[3] J. Connor, On strong matrix summability with respect to a modulus and statistical convergence, Canad. Math. Bull. 32 (1989) 194–198.
[4] J. Connor, Two valued measures and summability, Analysis 10 (1990) 373–385.
[5] J. Connor, J. Kline, On statistical limit points and the consistency of statistical convergence, J. Math. Anal. Appl. 197 (1996) 389–392.
[6] J. Connor, A topological and functional analytic approach to statistical convergence, in: Analysis of Divergence, Birkhauser, Boston, 1999, pp. 403–413.
[7] R.G. Cooke, Infinite Matrices and Sequence Spaces, McMillan, 1950.
[8] G. Das, Sublinear functionals and a class of conservative matrices, Bull. Inst. Math. Acad. Sinica 15 (1987) 89–106.
[9] K. Demirci, A-statistical core of a sequence, Demonstratio Math. 33 (2000) 343–353.
[10] K. Demirci, $\mathcal{I}$-limit superior and limit inferior, Math. Commun. 6 (2001) 165–172.
[11] H. Fast, Sur la convergence statistique, Colloq. Math. 2 (1951) 241–244.
[12] A.R. Freedman, J.J. Sember, Densities and summability, Pacific J. Math. 95 (1981) 293–305.
[13] J.A. Fridy, On statistical convergence, Analysis 5 (1985) 301–313.
[14] J.A. Fridy, Statistical limit points, Proc. Amer. Math. Soc. 118 (1993) 1187–1192.
[15] J.A. Fridy, C. Orhan, Statistical limit superior and limit inferior, Proc. Amer. Math. Soc. 125 (1997) 3625–3631.
[16] J.A. Fridy, C. Orhan, Statistical core theorems, J. Math. Anal. Appl. 208 (1997) 520–527.
[17] E. Kolk, Matrix summability of statistically convergent sequences, Analysis 13 (1993) 77–83.
[18] P. Kostyrko, M. Mačaj, T. Šalát, Statistical convergence and $\mathcal{I}$-convergence, Real Anal. Exchange, submitted for publication.
[19] P. Kostyrko, M. Mačaj, T. Šalát, $\mathcal{I}$-convergence, Real Anal. Exchange, submitted for publication.
[20] G.G. Lorentz, A contribution to the theory of divergent sequences, Acta Math. 80 (1948) 167–190.
[21] L. Loone, Knopp's core and almost convergency core in space m, Tartu Rikl. Il. Toimetised Vih. 335 (1975) 148–156.
[22] S.L. Mishra, B. Satapathy, Invariant means and σ-core, J. Pure Appl. Math. 60 (1994) 151–158.
[23] H.I. Miller, A measure theoretical subsequence characterization of statistical convergence, Trans. Amer. Math. Soc. 347 (1995) 1811–1819.
[24] H.I. Miller, C. Orhan, On almost convergent and statistically convergent subsequences, Acta Math. Hungar. 93 (2001) 135–151.
[25] I. Niven, H.S. Zuckerman, An Introduction to the Theory of Numbers, fourth ed., Wiley, New York, 1980.
[26] C. Orhan , Sublinear functionals and Knopp's core theorem, Internat. J. Math. Math. Sci. 3 (1990) 461–468.

[27] C. Orhan, S. Yardımcı, Banach and statistical cores of bounded sequences, Czechoslovak Math. J., submitted for publication.
[28] G.M. Petersen, Regular Matrix Transformations, McGraw–Hill, London, 1966.
[29] T. Šalát, On statistically convergent sequences of real numbers, Math. Slovaca 30 (1980) 139–150.
[30] P. Schaefer, Infinite matrices and invariant means, Proc. Amer. Math. Soc. 36 (1972) 104–110.
[31] S. Simons, Banach limits, infinite matrices and sublinear functionals, J. Math. Anal. Appl. 26 (1969) 640–655.